Chapter 2
Tribology of Machining

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and J. P. Davim

Abstract The chapter focuses on the tribological interactions in the area of machining and grinding and details the frictional interactions at the chip-tool interface. The chapter then discusses cutting, ploughing and sliding interaction models that are generally applied to the actions of cutting tools and provides solutions to reducing these interactions using solid and liquid lubricants. The authors intend to provide an understanding to readers of the chapter to design better processes for machining difficult-to-machine materials used in industry at large.

2.1 Frictional Interactions in Machining

Initial studies on chip-tool interactions during machining operations were carried out by Professor David Tabor and his team at the Cavendish Laboratory at the University of Cambridge in the United Kingdom during the late 1970s. In their initial studies, Doyle et al. [17] constructed a transparent sapphire cutting tool bonded to a tool holder that transmitted the action of chip formation so that it could
be observed. The reflection of the freshly cut chip is transmitted through the tool by reflecting the image on to a projection face that is highly polished. Doyle et al. [17] reported that they used pure lead and pure tin in air to witness the mechanism of metal transfer to the cutting tool and defined the nature of contact in terms of contact zones as the chip moved across the surface of the tool at low cutting speeds. Initially, two zones were noted, one of sliding across the rake face of the tool (zone 1) and one consisting of the chip material sticking to the rake face (zone 2) ahead of zone 1. On further inspection of the images obtained using a cine camera, zone 1 comprised of two sub zones, namely: zone 1a (where the chip material slides at the edge of the cutting tool on its rake face) and zone 1b (where the chip material sticks to the cutting edge). Further studies by Horne et al. [27], further characterized the nature of contact between the cut chip and the surface of the tool. In an effort to understand the mechanics of chip formation and lubrication, a series of experiments were developed to understand how cooling lubricants provide a thin film between the chip and tool material. In their studies, various lubricants were used and dropped into the chip-tool zone in an effort to provide the means of separation between tool and chip. The lubricant was shown to enter the chip at the side of the material and is absorbed beneath the chip as the chip moves across the rake face. The formation of bubbles is also noticeable when machining aluminium with CCl₄, and may significantly contribute to the change in the mechanism of material transfer from chip to tool or the mechanics of machining. The work currently performed at Purdue University is based on work that was previously conducted at the Cavendish Laboratory and focuses on quantifying the interactions between chip and tool. This work investigates the applicability of applying orthogonal and oblique cutting theories. The initial work was completed by Madhavan [44]. Again, the use of transparent sapphire tools was employed. The cutting tool has a highly polished surface and was used with a research apparatus that is similar in action to the configuration of a metal planer. A Newport slide, model number PM500-4, was used for the linear slide with which to move the workpiece toward the cutting tool. The slide is controlled by a microprocessor. The velocity of the workpiece ranged from 25 mm/s to 150 mm/s. The use of an angle plate was necessary to attach the workpiece to the slide. Another slide was used to adjust the depth of cut of the cutting tool. The entire apparatus was mounted to a vibration isolation table. Soda-lime glass tool was used for comparison with the sapphire tool in order to allow for a comparison of frictional constants and sliding mechanisms between the two substrates. These were again highly polished using small cuboids of soda-lime glass for the rough polish. The final polish involved the use of 1 μm diameter cerium oxide particles. The experiments were imaged using an Olympus model OM-4T microscope. The magnification ranged from 50X to 200X. Force measurements were made with a Kistler piezoelectric transducer. The transducer interestingly enough was configured between the back of the cutting tool and the actual tool holder. The signal produced by the transducer was recorded using an oscilloscope. This was capable of recording the cutting force and the thrust force during machining.
These experiments were conducted for both smooth, polished tools and roughened tools. This was performed to determine if the tool would replicate the rake face of a conventional tool. The tools reflect the image of the rake face to the side of the tool for easy observation as described by Doyle et al. [17]. The initial experiments involved the cutting of wax to develop a basic idea of what may occur. An observation of the process showed that microcracks were formed during the planing of wax. This in turn created a crack along the shear plane. This observation would begin the basis of machining other plastically deformable materials in order to observe the mechanism of machining.

The use of pure lead was considered essential for the experiments concerned with understanding rake face interactions. This was performed using the dry cutting and lubricated conditions. The lubricant was composed of a mixture of two parts of oil and one part of ink. From these experiments, it was determined that three distinct zones form. The three zones are: Ia, Ib, and II. In zone Ia, the chip interacts with the tool that is known as intimate sliding contact. In zones Ib and II, the chip experiences sticking and metal transfer to the rake face of the tool. When the lubricant is used, metal transfer does not occur in zones Ib and II. It was observed by Doyle et al. [17], that zone II does not form when the experiment is conducted in a vacuum. The idea that there are three distinct zones is puzzling if two of the zones experience the same sliding action. Perhaps two zones with one zone composed of two sub zones would have provided a better explanation of the situation. The magnitude of shear strain is briefly mentioned. The average shear strain for lead is 40–100. The shear strain for aluminium and copper is approximately 20. There were no calculations included for these values, nor a discussion about the effect of strain causing chip curl. Further experiments were conducted in the later 1990s by Ackroyd [1], who used a new piece of apparatus for characterizing chip-tool interactions. This work further investigated the frictional interactions initially noted by Madhavan [44]. The cutting tool used was again a highly polished sapphire tool bit.

The experimental workpieces used included pure lead as well as pure tin. Both of these materials were obtained from Goodfellow in England. The percentage purity of both materials was 99.95%. The dimensions of the workpiece were 50 mm square × 2 mm thick. Ackroyd [1] also investigated the machining of brass and pure aluminium. Sapphire tools were compared to aluminium and high-speed steel tools. With the use of these tools, a comparative analysis was undertaken to determine if the frictional effects were similar for all tool materials. The sapphire tool was once again polished with cerium oxide and then cleaned with acetone. The rake angles of the tools were 10° and −5°. The experimental apparatus uses a linear drive made by Anorad (model number LW5-750). This unit is capable of a maximum velocity of 2 m/s. The workpiece is similarly mounted to this unit to achieve the maximum workpiece velocity. The vertical stage used was the Newport 433 model. This stage is equipped with the Newport DMH-1, which is a digital micrometer. This allows for measurement of a 1 μm resolution over a 15 mm range. The Newport stage is attached to a Kistler 9254 dynamometer. This is attached to cast iron v-blocks that have been mounted to a worktable. The
Kistler unit is used in conjunction with a dual mode amplifier (Kistler 5010B). The optics used for observation can magnify up to 200X. The CCD camera used is a Sony DXC-930. The camera was used in conjunction with s-VHS video. The model used was a Panasonic AG-1970 with a capability of 30 frames per second. The digital high-speed camera used was a Kodak Motion Corder Analyzer Sr-Ultra. It is difficult to determine more than two zones resulting from the machining experiment. For the initial experiments, the following are the parameters were used: The depth of cut was 200 μm; width of cut was 2 mm; length of cut was 50 mm, and the cutting speed ranged from 0.5 mm/sec to 500 mm/sec. It was discovered, and was confirmed by Robinson [56], that the cutting edge plays a significant role in the machining experiments. As the nose radius becomes larger, the depth of cut determines whether the cutting tool cuts, or shears the workpiece material. A large cutting nose radius can create a negative rake angle on the cutting tool if the depth of cut is smaller than the nose radius.

From this work, it can be seen that there are two zones. The first zone involves the chip sliding with no deposit of metal. The second zone involves the chip sticking with some material deposited onto the tool. It was observed that a blunt tool has a higher normal force than a sharp tool. The frictional force versus normal force ratio decreases with the increase in velocity. It was also noted that there were no material deposits to the rake face at higher velocities. The discussion and conclusions of this work show that continual frictional forces increase. This is not a claim that can be substantiated. If frictional force continually increases with friction, it would reach a catastrophic point where the tool would fail. The conclusion was made using a single machining force measurement to examine both the sticking and sliding regions in question. Both zones cannot be observed simultaneously, therefore a direct comparison cannot be made.

Subsequent work completed by Hwang [29] followed closely the work of Ackroyd [1]. The apparatus that was previously used was upgraded with new components. The use of sapphire and high-speed tools for machining was investigated further by Hwang [29]. The experiments were designed to investigate the sliding and sticking zones. The apparatus was upgraded with a new linear slide. The linear slide was ball screw driven and was supplied by Parker as model number ERB80-B02LAJX-GXS677-A96. This allows for a larger power motor to drive the linear slide. The increased power allowed a larger variety of materials to be investigated. The slide has a maximum speed of 750 mm/sec. Again, the vertical stage used was a Newport model number 433. This is actuated using the Newport digital micrometer model number DMH-1. The imaging system remained the same as that used by Ackroyd [1].

A wider variety of plastically deformed materials were used in the experiments. The list includes oxygen free, high conductivity copper (OFHC), cartridge brass, pure lead, Al 1100 aluminium, and Al 6061-T6 aluminium. Hardness values were taken for all materials except for pure lead. The following are the Vickers’ hardness values: OFHC = 89.7 kg mm$^{-2}$, cartridge brass = 153 kg mm$^{-2}$, Al 1100 = 49.5 kg mm$^{-2}$, and Al 6061-T6 = 116 kg mm$^{-2}$. The workpiece specimens investigated closely were pure lead and Al 6061-T6. The depth of cut for the
lead was 200 μm, while the depth of cut for the 6061 was 100 μm. The cutting velocity spanned from 0.5 mm/s to 500 mm/s. The exact speeds used were 0.5 mm/s, 5 mm/s, 50 mm/s, and 500 mm/s, respectively. These experiments looked at the effects of lubrication on the chip formation process. The use of lubricant was observed to reduce the length of chip-tool contact length. The investigation focused on the rake face’s sliding and sticking regions. This is a continuation of the work performed by Ackroyd [1] and shows the variation in machining different materials. Another aspect of the chip formation process was closely examined. The secondary deformation in a chip was explored with the use of a quick-stop experiment. This is an experiment that stops the cutting tool about three-quarters of the way across the workpiece. This will allow for the examination of the primary shear zone and grain orientation in the chip compared to the substrate. Another segment of the research investigated the use of modulation during machining pure metals with lubricant. The experiment was conducted with a vertical slide as opposed to a horizontal slide. It was not directly observed that the cutting fluid penetrated into the intermittent contact zone during these experiments. This assumption is made due to chip debris remaining static and the elimination of the metal deposit zone. During modulation, frictional forces are much smaller. It was noted that cutting remains under high pressure in the gap ahead of the cutting tool and that the cutting fluid reduced the region of the chip-tool contact. In this region, the contact length reduction promotes a reduction in frictional force. The application of cutting fluid will cause the zone of metal deposits to move further away from the tool edge. This will prevent the expansion of the stagnant metal zone. This is beneficial to eliminate a partial frictional constant. However, it is questionable as to whether modulation is solely responsible for this action.

The measurement of the rake face temperature was performed using an infrared imaging system. The signal is reflected at the back of the tool in order to investigate the temperature of the rake face of the tool. The methods of calculating temperature developed by Rapier, Boothroyd, and Loewen and Shaw were used to calculate the rake face temperature. These calculations were then compared to the measured data. From these results, it can be reasoned that modulation does not solve the frictional contact problem. However, modulation does create a constant cutting length and constant values of friction between chip and tool. However, this action alone is dependent upon the stroke length chosen, and will be different for each material machined and the corresponding depth of cut.

The research conducted by Lee [40] follows closely the work of both Ackroyd [1] and Hwang [29]. Lee’s study included using particle image velocimetry to measure chip velocity. This utilized a linear slide with a ball screw. This allows for an adjustment of the depth of cut as little as 1 μm. A charge-coupled device (CCD) was used for high-speed imaging. Similar to the Hwang’s work, a Kodak Motion Coder Analyzer Sr-Ultra is used. An optical microscope records the chip formation process, and employs a Nikon Optiphot that can examine the specimen up to 200X magnification. The CCD can capture images of up to 10,000 frames per second using a black and white format. The spatial resolution is 3.3 μm pixel size. The
experiments were conducted dry. The workpiece specimens used were commercially pure lead and copper that is 99.95% pure. These were both obtained from Goodfellow, UK. In the initial investigation using 6061-T6 aluminium, a built-up edge (BUE) was observed on the rake face. The BUE changes the rake angle thus increasing the chip velocity more than in the machining of other materials.

The velocity of the chip was inspected using constrained workpiece specimens. These areas show the rake face and the side view of the cut chip. This allows for the investigation of the metal deposit on the rake face and the effects of deformation in the secondary zone. A vertical stage moves the tool into contact with the workpiece. This is made possible through the use of a micrometer. At the foundation is the ball screw drive that brings the workpiece into contact with the cutting tool. This is a satisfactory experimental apparatus. However, there remains much to be desired with this particular apparatus in terms of rigidity and range of workpiece cutting speeds.

2.2 Cutting, Ploughing and Sliding Interactions (after [31])

The nature of the contact between surfaces is an important aspect of understanding the function of tribology in machining [31] and hereafter). The properties of the materials in contact are homogeneous and isotropic. Macrocontact conditions are most useful in models for friction when there is lubrication and the effects of surface heterogeneities are of little importance. Hertz’s equations allow engineers to calculate the maximum compressive contact stresses and contact dimensions for non-conforming bodies in elastic contact. The parameters required to calculate the quantities and the algebraic equations used for simple geometries are given in Table 2.1 [65]. It should be noted that Hertz’s equations apply to static, or quasi-static, elastic cases. In the case of sliding, plastic deformation, contact of very rough surfaces, or significant fracture, both the distribution of stresses and the contact geometry will be altered. Hertz’s contact equations have been used in a range of component design applications, and in friction and wear models in which the individual asperities are modeled as simple geometric contacts.

Greenwood and Williamson [20] developed a surface geometry model that modeled contacts as being composed of a distribution of asperities. From that assumption, contact between such a surface and a smooth, rigid plane could be determined by three parameters: the asperity radius ($R$), the standard deviation of asperity heights ($\sigma^*$), and the number of asperities per unit area. To predict the extent of the plastic deformation of asperities, the plasticity index ($\psi$), also a function of the hardness ($H$), elastic modulus ($E$), and Poisson’s ratio ($\nu$), was introduced.

$$\psi = \left( \frac{E'}{H} \right) \left( \frac{\sigma^*}{R} \right)^{1/2}$$ (2.1)
Table 2.1 Equations for calculating elastic (Hertz) contact stress [31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Contact dimension</th>
<th>Contact stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere-on-flat</td>
<td>( a = 0.721\sqrt{PDE^*} )</td>
<td>( S_c = 0.918\sqrt{P/(D^2E^*)^2} )</td>
</tr>
<tr>
<td>Cylinder-on-flat</td>
<td>( b = 1.6\sqrt{pDE^*} )</td>
<td>( S_c = 0.798\sqrt{p/DE^*} )</td>
</tr>
<tr>
<td>Cylinder-on-cylinder (axes parallel)</td>
<td>( b = 1.6\sqrt{pE^*/A} )</td>
<td>( S_c = 0.798\sqrt{pA/E^*} )</td>
</tr>
<tr>
<td>Sphere in a spherical socket</td>
<td>( a = 0.721\sqrt{PE^*/B} )</td>
<td>( S_c = 0.918\sqrt{[3]P(B/E^*)^2} )</td>
</tr>
<tr>
<td>Cylinder in a circular groove</td>
<td>( b = 1.6\sqrt{pE^*/B} )</td>
<td>( S_c = 0.798\sqrt{pB/E^*} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>Normal force</td>
</tr>
<tr>
<td>( p )</td>
<td>Normal force per unit contact length</td>
</tr>
<tr>
<td>( E_{1,2} )</td>
<td>Modulus of elasticity for bodies 1 and 2, respectively</td>
</tr>
<tr>
<td>( \nu_{1,2} )</td>
<td>Poisson’s ratios for bodies 1 and 2, respectively</td>
</tr>
<tr>
<td>( D )</td>
<td>Diameter of the curved body, if only one if curved</td>
</tr>
<tr>
<td>( D_{1,2} )</td>
<td>Diameters of bodies 1 and 2, where ( D_1 &gt; D_2 ) by convention</td>
</tr>
<tr>
<td>( S_c )</td>
<td>Maximum compressive stress</td>
</tr>
<tr>
<td>( a )</td>
<td>Radius of the elastic contact</td>
</tr>
<tr>
<td>( b )</td>
<td>Width of a contact (for cylinders)</td>
</tr>
<tr>
<td>( E^* )</td>
<td>Composite modulus of bodies 1 and 2</td>
</tr>
<tr>
<td>A, B</td>
<td>Functions of the diameters of bodies 1 and 2</td>
</tr>
</tbody>
</table>

Where, \( E' = E/(1 - \nu^2) \). This basic formulation was refined by various investigators such as Whitehouse and Archard [64] to incorporate other forms of height distributions, and the incorporation of a distribution of asperity radii, represented by the correlation distance \( \beta^* \), which produced higher contact pressures and increased plastic flow. Therefore,

\[
\psi = \left( \frac{E'}{H} \right) \left( \frac{\sigma^*}{\beta} \right)^{1/2}
\]  

(2.2)

Hirst and Hollander [25] used the plasticity index to develop diagrams to predict the start of scuffing wear. Other parameters, such as the average or root mean square slope of asperities, have been incorporated into wear models to account for such peculiarities [43]. Worn surfaces are observed to be much more complex than simple arrangements of spheres, or spheres resting on flat planes, and Greenwood readily acknowledged some of the problems associated with simplifying assumptions about surface roughness [21]. A comprehensive review of surface texture measurement methods have been given by Song and Vorburger [59]. The most commonly used roughness parameters are listed in Table 2.2. Parameters such as skewness are useful for determining lubricant retention qualities of surfaces, since they reflect the presence of cavities. However, one parameter alone cannot precisely model the geometry of surfaces. It is possible to have the same average roughness (or RMS roughness) for two different surfaces.
Small amounts of wear can change the roughness of surfaces on the microscale and disrupt the nanoscale structure as well. Some of the following quantities have been used in models for friction:

1. The true area of contact;
2. The number of instantaneous contacts comprising the true area of contact;
3. The typical shapes of contacts (under load);
4. The arrangement of contacts within the nominal area of contact; and
5. The time needed to create new points of contact.

Finally, contact geometry-based models for friction generally assume that the normal load is constant. This assumption may be unjustified, especially when sliding speeds are relatively high, or when there are significant friction and vibration interactions in the tribosystem. As the sliding speed increases, frictional heating increases and surface thermal expansion can cause intermittent contact. The growth and excessive wear of intermittent contact points is termed thermoelastic instability (TEI) [12]. TEI is only one potential source of the interfacial dynamics responsible for stimulating vibrations and normal force variations in sliding contacts. Another major cause is the eccentricity of rotating shafts, run-out, and the transmission of external vibrations. Static friction and stick–slip behavior are considered, and as with kinetic friction, the causes for such phenomena can be interpreted on several scales.

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**Table 2.2** Definitions of surface roughness parameters [31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic average roughness</td>
<td>( R_a = \frac{1}{N} \sum_{i=1}^{N}</td>
</tr>
<tr>
<td>Root-mean-square roughness</td>
<td>( R_q = \left[ \frac{1}{N} \sum_{i=1}^{N} y_i^2 \right]^{1/2} )</td>
</tr>
<tr>
<td>Skewness</td>
<td>( R_{sk} = \frac{1}{N R_{q}^{3}} \sum_{i=1}^{N} y_i^3 )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>( R_{kurtosis} = \frac{1}{N R_{q}^{4}} \sum_{i=1}^{N} y_i^4 )</td>
</tr>
</tbody>
</table>

Let \( y_i \) = vertical distance from the \( i \)th point on the surface profile to the mean line

Thus, the following are defined:

- **Arithmetic average roughness**
  \[ R_a = \frac{1}{N} \sum_{i=1}^{N} |y_i| \]

- **Root-mean-square roughness**
  \[ R_q = \left[ \frac{1}{N} \sum_{i=1}^{N} y_i^2 \right]^{1/2} \]

- **Skewness**
  \[ R_{sk} = \frac{1}{N R_{q}^{3}} \sum_{i=1}^{N} y_i^3 \]

  A measure of the symmetry of the profile
  \[ R_{sk} = 0 \] for a Gaussian height distribution

- **Kurtosis**
  \[ R_{kurtosis} = \frac{1}{N R_{q}^{4}} \sum_{i=1}^{N} y_i^4 \]

  A measure of the sharpness of the profile
  \[ R_{kurtosis} = 3.0 \] for a Gaussian height distribution
  \[ R_{kurtosis} < 3.0 \] for a broad distribution of heights
  \[ R_{kurtosis} > 3.0 \] for a sharply-peaked distribution

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2.2.1 Static Friction and Stick–slip Phenomena

If all possible causes for friction are to be considered, it is reasonable to find out whether there are other means to cause bodies to stay together without the requirement for molecular bonding. Surfaces may adhere, but adherence is not identical to adhesion, because there is no requirement for molecular bonding. If a certain material is cast between two surfaces and, after penetrating and filling irregular voids in the two surfaces, solidifies to form a network of interlocking contacting points there may be strong mechanical joint produced, but no adhesion. Adhesion (i.e., electrostatically balanced attraction/chemical bonding) in friction theory meets the need for an explanation of how one body can transfer shear forces to another. Clearly, it is convenient to assume that molecular attraction is strong enough to allow the transfer of force between bodies, and in fact this assumption has led to many of the most widely used friction theories. From another perspective, is it not equally valid to consider that if one pushes two rough bodies together so that asperities penetrate, and then attempts to move those bodies tangentially, the atoms may approach each other closely enough to repel strongly, thus causing a backlash against the bulk materials and away from the interface. The repulsive force parallel to the sliding direction must be overcome to move the bodies tangentially, whether accommodation occurs by asperities climbing over one another, or by deforming one another. In the latter, it is repulsive forces and not adhesive bonding that produces sliding resistance. This section focuses on static friction and stick slip phenomena.

Ferrante et al. [19] have provided a comprehensive review of the subject. A discussion of adhesion and its relationship to friction has been conducted by Buckley [11]. Atomic probe microscopes permit investigators to study adhesion and lateral forces between surfaces on the atomic scale. The force required to shift the two bodies tangentially must overcome bonds holding the surfaces together. In the case of dissimilar metals with a strong bonding preference, the shear strength of the interfacial bonds can exceed the shear strength of the weaker of the two metals, and the static friction force \( F_s \) will depend on the shear strength of the weaker material \( (\tau_m) \) and the area of contact \( (A) \). In terms of the static friction coefficient \( \mu_s \),

\[
F_s = \mu_s P^* = \tau_m A
\]  

(2.3)

Or,

\[
\mu_s = \left( \frac{\tau_m}{P^*} \right) A
\]  

(2.4)

where \( P^* \), the normal force is comprised of the applied load and the adhesive contribution normal to the interface. Under specially controlled conditions, such as friction experiments with clean surfaces in vacuum, the static friction coefficients can be greater than 1.0, and the experiment becomes a test of the shear strength of the solid materials than of interfacial friction. Scientific understanding and
approaches to modeling friction has been strongly influenced by concepts of solid surfaces and by the instruments available to study them. Atomic-force microscopes and scanning tunneling microscopes permit views of surface atoms with high resolution and detail. Among the first to study nanocontact frictional phenomena were McClellan et al. [41, 42]. A tungsten wire with a very fine tip is brought down to the surface of a highly oriented, cleaved basal plane of pyrolytic graphite as the specimen is oscillated at 10 Hz using a piezoelectric driver system. The cantilevered wire is calibrated so that its spring constant is known (2500 N/m) and the normal force could be determined by measuring the deflection of the tip using a reflected laser beam. As the normal force is decreased, the contributions of individual atoms to the tangential force became apparent. At the same time, it appeared that the motion of the tip became less uniform, exhibiting atomic-scale stick–slip.

Thompson and Robbins [61] discussed the origins of nanocontact stick–slip when analyzing the behavior of molecularly thin fluid films trapped between flat surfaces of face-centered cubic solids. At that scale, stick–slip was believed to arise from the periodic phase transitions between ordered static and disordered kinetic states. Immediately adjacent to the surface of the solid, the fluid assumed a regular, crystalline structure, but this was disrupted during each slip event. The experimental data points of friction force per unit area versus time exhibited extremely uniform classical stick–slip appearance. Once slip occurred, all the kinetic energy must be converted into potential energy in the film. In subsequent papers (Robbins et al. [54, 55] this group of authors used this argument to calculate the critical velocity, $v_c$, below which the stick–slip occurs is:

$$v_c = c(\sigma F_s/M)^{1/2}$$

(2.5)

where $\sigma$ is the lattice constant of the wall, $F_s$ is the static friction force, $M$ is the mass of the moving wall, and $c$ is a constant.

Friction is defined as the resistance to relative motion between two contacting bodies parallel to a surface that separates them. Motion at the atomic scale is unsteady. In nanocontact, accounting for the tangential components of thermal vibrations of the atoms thus affects our ability to clearly define relative motion between surfaces. Under some conditions it may be possible to translate the surface laterally while the adhesive force between the probe tip and the opposite surface exceeds the externally applied tensile force. Landman et al. [39] reviewed progress in the field of molecular dynamics (MD). By conducting MD simulations of nickel rubbing a flat gold surface, Landman illustrated how the tip can attract atoms from the surface simply by close approach without actual indentation. A connective neck or bridge of surface atoms was observed to form as the indenter was withdrawn. The neck can exert a force to counteract the withdrawal force on the tip, and the MD simulations clearly model transfer of material between opposing asperities under pristine surface conditions. Landman has subsequently conducted numerous other MD simulations, including complete indentation and indentation in the presence of organic species between the indenter and substrate. Belak and Stowers [4], using a
material volume containing 43,440 atoms in 160 layers, simulated many of the deformational features associated with metals, such as edge dislocations, plastic zones, and point defect generation. Calculated shear stresses for a triangular indenter passing along the surface exhibited erratic behavior, not unlike that observed during metallic sliding under clean conditions. Pollock and Singer [48] compiled a series of papers on atomic-scale approaches to friction.

While MD simulations and atomic-scale experiments continue to provide fascinating insights into frictional behavior, under idealized conditions, most engineering tribosystems are non-uniform. Not only are surfaces not atomically flat, but the materials are not homogeneous, and surface films and contaminant particles of many kinds, much larger than the atomic scale, may influence interfacial behavior. Static friction coefficients measured experimentally under ambient or contaminated conditions probably will not assume the values obtained in controlled environments. In a series of carefully conducted experiments on the role of adsorbed oxygen and chlorine on the shear strength of metallic junctions, Wheeler [63] showed how, \( \mu_s \), can be reduced in the presence of adsorbed gases. On the other hand, static friction coefficients for pure, well-cleaned metal surfaces in the presence of non-reactive gases like He can be relatively high. It is interesting to note that the friction of copper on nickel and the friction of nickel on copper are quite different. This is not an error, but rather a demonstration of the fact that reversing the materials of the sliding specimen and the counterface surface can affect the measured friction, confirming the assertion that friction is a property of the tribosystem and not of the materials in contact. A cryotribometer was used to obtain the data in Table 2.3. The length of time that two solids are in contact can also affect the relative role that adhesion plays in establishing the value of the static friction coefficient. Two distinct possibilities can occur: (a) if the contact becomes contaminated with a lower shear-strength species, the friction will decline; and (b) if the contact is clean and a more tenacious interfacial bond develops, the static friction will tend to increase. Akhmatov [2] demonstrated that by using cleaved rock salt that the formation of surface films over time lowers static friction. The opposite effect has been demonstrated for metals. A first approximation of rising static friction behavior is given by,

<table>
<thead>
<tr>
<th>Material combination</th>
<th>300 K</th>
<th>80 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe (99.9 %) on Fe (99.99 %)</td>
<td>1.09</td>
<td>1.04</td>
</tr>
<tr>
<td>Al (99 %) on Al (99 %)</td>
<td>1.62</td>
<td>1.60</td>
</tr>
<tr>
<td>Cu (99.95 %) on Cu (99.95 %)</td>
<td>1.76</td>
<td>1.70</td>
</tr>
<tr>
<td>Ni (99.95 %) on Ni (99.95 %)</td>
<td>2.11</td>
<td>2.00</td>
</tr>
<tr>
<td>Au (99.98 %) on Au (99.98 %)</td>
<td>1.88</td>
<td>1.77</td>
</tr>
<tr>
<td>Ni (99.95 %) on Cu (99.95 %)</td>
<td>2.34</td>
<td>2.35</td>
</tr>
<tr>
<td>Cu (99.95 %) on Ni (99.95 %)</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Au (99.98 %) on Al (99 %)</td>
<td>1.42</td>
<td>1.50</td>
</tr>
<tr>
<td>Fe (99.9 %) on Cu (99.95 %)</td>
<td>1.99</td>
<td>2.03</td>
</tr>
</tbody>
</table>
\[ \mu_s(t) = \mu_{S(t=\infty)} - \left[ \mu_{S(t=\infty)} - \mu_{S(t=0)} \right] e^{-ut} \]  

(2.6)

where, \( \mu_s(t) \), is the current value of the static friction coefficient at time \( t \), \( \mu_s(t = \infty) \) is the limiting value of the static friction coefficient at long times, \( \mu_s(t = 0) \) is the initial static friction coefficient, and \( u \) is a rate constant. In contrast to exponential dependence on time, Buckley showed that by using data for tests of single-crystal Au touching Cu-5 % Al alloy that junction growth can cause the adhesive force to increase linearly with time.

When materials are placed in intimate contact, it is not unexpected that the atoms on their surfaces will begin to interact. The degree of this interaction will depend on the contact pressure, temperature, and the degree of chemical reactivity that the species have for each other, hence, static friction can change with the duration of contact. Despite the two opposite dependencies of static friction on time of contact, observations are consistent from a thermodynamic standpoint. Systems tend toward the lowest energetic state. In the case of interfaces, this state can be achieved either by forming bonds between the solids, or by forming bonds with other species (adsorbates and films) in the interface. The former process tends to strengthen the shear strength of the system, and the latter tends to weaken it. Sikorski [58] reported the results of experiments designed to compare friction coefficients of metals with their coefficients of adhesion (defined as the ratio of the force needed to break the bond between two specimens to the force which initially compressed them together). Rabinowicz [51] conducted a series of simple, tilting-plane tests with milligram- to kilogram-sized specimens of a variety of metals. Results demonstrated the static friction coefficient to increase as slider weight (normal force) decreased. For metal couples such as Au/Rh, Au/Au, Au/Pd, Ag/Ag, and Ag/Au, as the normal force increased over about six orders of magnitude (1 mg–1 kg), the static friction coefficients tended to decrease by nearly one order of magnitude.

Under low contact pressures, surface chemistry effects can play a relatively large role in governing static friction behavior. However, under more severe contact conditions, such as extreme pressures and high temperatures, other factors, more directly related to bulk properties of the solids, dominate static friction behavior. When very high pressures and temperatures are applied to solid contacts, diffusion bonds or solid-state welds can form between solids, and the term static friction ceases to be applicable. Table 2.4 lists a series of reported static friction coefficients. Note that in certain cases, the table references list quite different values for these coefficients. The temperature of sliding contact can affect the static friction coefficient. This behavior was demonstrated for single crystal ceramics by Miyoshi and Buckley [45], who conducted static friction tests of pure iron sliding on cleaned \{0001\} crystal surfaces of silicon carbide in a vacuum \((10^{-8} \text{ Pa})\). For both \(< 1010 >\) and \(< 1120 >\) sliding directions, the static friction coefficients remained about level (0.4 and 0.5, respectively) from room temperature up to about 400 °C; then they each rose by about 50 % as the temperature rose to 800 °C. The authors attributed this effect to increased adhesion and plastic flow.
The role of adsorbed films on static friction suggests that one effective strategy for alleviating or reducing static friction is to introduce a lubricant or other surface treatment to impede the formation of adhesive bonds between mating surfaces. Contamination of surfaces from exposure to the ambient environment performs essentially the same function, but is usually less reproducible. Campbell [14] demonstrated how the treatment of metallic surfaces by oxidation can reduce the static friction coefficient. Oxide films were produced by heating metals in air. Sulfide films were produced by immersing the metals in sodium sulfide solution. Except for the film on steel, film thicknesses were estimated to be 100–200 nm. Results from ten experiments, using a three ball-on-flat plate apparatus, were averaged to obtain static friction coefficients. In addition to producing oxides and sulfides, Campbell also tested oxide and sulfide films with Acto oil. The results of this investigation are shown in Table 2.5. For copper, the static friction coefficient ($\mu_s = 1.21$, with no film) decreased when the sulfide film thickness was increased from 0 to about 300 nm, after which the static friction coefficient remained about constant at 0.66.

### Table 2.4 Static friction coefficients for metals and non-metals (Dry or unlubricated conditions) [31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Material combination</th>
<th>Fixed specimen</th>
<th>Moving specimen</th>
<th>$\mu_s$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>Aluminium</td>
<td>1.05</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td>Steel, mild</td>
<td>Steel, mild</td>
<td>0.61</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td>Titanium</td>
<td>Titanium</td>
<td>0.54</td>
<td>[68]</td>
<td></td>
</tr>
<tr>
<td>Al, 6061-T6</td>
<td>Al, 6061-T6</td>
<td>0.42</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>Copper</td>
<td>0.28</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Steel, 1032</td>
<td>Steel, 1032</td>
<td>0.35</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4 V</td>
<td>Ti-6Al-4 V</td>
<td>0.34</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>Cast iron</td>
<td>1.05</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>Cast iron</td>
<td>0.4</td>
<td>[67]</td>
<td></td>
</tr>
<tr>
<td>Steel, hardened</td>
<td>Steel, hardened</td>
<td>0.78</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Babbitt</td>
<td>0.42, 0.70</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Graphite</td>
<td>0.21</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td>Steel, mild</td>
<td>Steel, mild</td>
<td>0.74</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lead</td>
<td>0.95</td>
<td>[66]</td>
<td></td>
</tr>
<tr>
<td>Steel, 1032</td>
<td>Aluminium</td>
<td>0.47</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>0.32</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steel, 1032</td>
<td>0.31</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ti-6Al-4 V</td>
<td>0.36</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Steel, stainless 304</td>
<td>Copper</td>
<td>0.33</td>
<td>[69]</td>
<td></td>
</tr>
<tr>
<td>Tin</td>
<td>Iron</td>
<td>0.55</td>
<td>[68]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tin</td>
<td>0.74</td>
<td>[68]</td>
<td></td>
</tr>
<tr>
<td>Titanium</td>
<td>Aluminium</td>
<td>0.54</td>
<td>[68]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Titanium</td>
<td>0.55</td>
<td>[68]</td>
<td></td>
</tr>
</tbody>
</table>
The extent to which the solid lubricant can reduce static friction may be dependent on temperature, as confirmed by Hardy’s earlier studies on the static friction of palmitic acid films on quartz. Between 20 and 50 °C the static friction coefficient decreases until melting occurs, at which time the lubricant loses its effectiveness.

Stick-slip is often referred to as a relaxation–oscillation phenomenon, and consequently, some degree of elasticity is needed in the sliding contact in order for stick–slip to occur. Israelachvili [30] considered stick–slip on a molecular level, as measured with surface forces apparatus. He considers the order–disorder transformations described by Thompson and Robbins [54, 61] in terms of simulations. Most classical treatments of stick–slip take a mechanics approach, considering that the behavior in unlubricated solid sliding is caused by forming and breaking adhesive bonds.

Stick-slip behavior can be modeled in several ways. Generally, the system is represented schematically as a spring-loaded contact, sometimes including a dashpot element to account for viscoelastic response [46]. The effects of time-dependent material properties on stick–slip behavior of metals is provided by Kosterin and Kragelski [34] and Kragelski [35]. Bowden and Tabor’s analysis [8] considers a free surface of inertial mass $m$ being driven with a uniform speed $\mathbf{v}$ in the positive $x$ direction against an elastic constant $k$. Then the instantaneous resisting force $F$ over distance $x$ equals $-kx$. With no damping of the resultant oscillation,

$$ma = -kx$$ (2.7)

where acceleration $a = (d^2x/dt^2)$. The frequency $n$ of simple harmonic motion is given by

$$n = (1/2\pi)(k/m)^{1/2}$$ (2.8)

Under the influence of a load $\mathbf{P}$ (mass $\mathbf{W}$ acting downward with the help of gravity $g$), the static friction force $F_s$ can be represented as

$$F_s = \mu_s \mathbf{P}$$ (2.9)

In terms of the deflection at the point of slip ($x$),

$$x = F_s/k$$ (2.10)

If the kinetic friction coefficient $\mu$ is assumed to be constant during slip, then

---

Table 2.5  Reduction of static friction by surface films [31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Material combination</th>
<th>$\mu_s$, No film</th>
<th>$\mu_s$, Oxide film</th>
<th>$\mu_s$, Sulfide film</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper-on-copper</td>
<td>1.21</td>
<td>0.76</td>
<td>0.66</td>
</tr>
<tr>
<td>Steel on steel</td>
<td>0.78</td>
<td>0.27</td>
<td>0.39</td>
</tr>
<tr>
<td>Steel on steel</td>
<td>0.78</td>
<td>0.19$^a$</td>
<td>0.16$^a$</td>
</tr>
</tbody>
</table>

$^a$ film and oil

---
\[ ma - \mu P = -kx \] (2.11)

Letting time \( t = 0 \) at the point of slip (where \( x = \frac{F_s}{k} \)), and the forward velocity \( v \ll \) the velocity of slip, then,

\[ x = \left(\frac{P}{k}\right)\left(\mu_s - \mu\right) \cos \omega t + \mu \] (2.12)

where, \( \omega = \sqrt{(k/m)0.12} \). In this case, the magnitude of slip, \( \delta \), is

\[ \delta = \frac{P(2\mu_s - 2\mu)}{k} \] (2.13)

From this equation, the larger the \( \mu \) relative to \( \mu_s \), the less the effects of stick–slip, and when they are equal, the sliding becomes completely steady. Kudinov and Tolstoy [37] derived a critical velocity above which stick–slip could be suppressed. This critical velocity \( \nu_c \) was directly proportional to the difference in the static and kinetic friction coefficients \( \Delta \mu \) and inversely proportional to the square root of the product of the relative dissipation of energy during oscillation \( (\psi = 4\pi \tau) \), the stiffness of the system \( k \), and the slider mass \( m \). Thus,

\[ \nu_c = \frac{\Delta \mu N}{\sqrt{\psi km}} \] (2.14)

where \( N \) is the factor of safety. The authors report several characteristic values of \( \Delta \mu \) for slideways on machine tools: cast iron on cast iron = 0.08, steel on cast iron = 0.05, bronze on cast iron = 0.02, and PTFE on cast iron = 0.04.

System resonance within limited stick–slip oscillation ranges was discussed by Bartenev and Lavrentev [3], who cited experiments in which an oscillating normal load was applied to a system in which stick–slip was occurring. The minimum in stick–slip amplitude and friction force occurred over a range of about 1.5–2.5 kHz, the approximate value predicted by \((1/2\pi)(k/m)^{-1/2}\). Rabinowicz [50] suggested two possible solutions:

1. Decrease the slip amplitude or slip velocity by increasing contact stiffness, increasing system damping, or increasing inertia; and
2. Lubricate or otherwise form a surface film to ensure a positive \( \mu \) versus velocity relationship.

The latter solution requires that effective lubrication be maintained, and stick–slip can return if the lubricant becomes depleted. The fact that stick–slip is associated with a significant difference between static and kinetic friction coefficients suggests that strategies that lower the former or raise the latter can be equally effective.

### 2.2.2 Sliding Friction

Sliding friction plays a very important role in many manufacturing processes. Sliding friction models, other than empirical models, can generally be grouped into five categories:
1. Plowing and cutting-based models;
2. Adhesion, junction-growth, and shear models;
3. Single- and multiple-layer shear models;
4. Debris layer and transfer layer models; and
5. Molecular dynamics’ models.

Each type of model was developed to explain frictional phenomena. Some of the models are based on observations that contact surfaces contain grooves that are suggestive of a dominant contribution from plowing. Single-layer models rely on a view of the interface showing flat surfaces separated by a layer whose shear strength controls friction. Some models involve combinations, such as adhesion plus plowing. Recent friction models contain molecular-level phenomena. Lubrication-oriented models and the debris-based models describe phenomena that take place in zone I, whereas most of the classical models for solid friction concern zone II phenomena. There are few models that take into account the effects of both the interfacial properties and the surrounding mechanical systems such as zone III models.

2.2.2.1 Models for Sliding Friction

Sliding friction models are summarized in this section of the chapter and fall into one, or more, of the five categories explained in the previous section.

(a) Plowing Model. Plowing models assume that the dominant contribution to friction is the energy required to displace material ahead of a rigid protuberance or protuberances moving along a surface. One of the simplest models for plowing is that of a rigid cone of slant angle \( \theta \) plowing through a surface under a normal load \( P \) [50]. If we assign a groove width \( w \) (i.e., twice the radius \( r \) of the circular section of the penetrating cone at surface level), the triangular projected area, \( A_p \), swept out as the cone moves along is as follows:

\[
A_p = \frac{1}{2} w (r \tan \theta) = \frac{1}{2} (2r) (r \tan \theta) = r^2 \tan \theta \]

The friction force \( F_p \) for this plowing contribution to sliding is found by multiplying the swept-out area by the compressive strength \( p \). Thus, \( F_p = (r^2 \tan \theta) \, p \), and the friction coefficient, if this were the only contribution, is \( \mu_p = \frac{F_p}{P} \). From the definition of the compressive strength \( p \) as force per unit area, we can write:

\[
p = \frac{P}{\pi r^2} \]

And

\[
\mu_p = \frac{F_p}{P} = \frac{(r^2 \tan \theta) p}{\pi r^2 p} = \frac{(\tan \theta)}{\pi} \]

82 M. J. Jackson et al.
This expression can also be written in terms of the apex angle of the cone \( \alpha \) (= 90 \(- \theta \)):

\[
\mu_p = \frac{2 \cot \alpha}{\pi}
\]  

(2.18)

Note that the friction coefficient calculated is for the plowing of a hard asperity and is not necessarily the same as the friction coefficient of the material sliding along the sides of the conical surface. Table 2.6 shows the maximum plowing contribution to friction for various metals.

(b) Adhesion, Junction Growth, and Shear (AJS) Models. The AJS interpretations of friction are based on a scenario in which two rough surfaces are brought close together, causing the highest peaks (asperities) to touch. As the normal force increases, the contact area increases and the peaks are flattened. Asperity junctions grow until they are able to support the applied load. Adhesive bonds form at the contact points. When a tangential force is applied, the bonds must be broken, and overcoming the shear strength of the bonds results in the friction force. Early calculations comparing bond strengths to friction forces obtained in experiments raised questions as to the general validity of such models. Observations of material transfer and similar phenomena suggested that the adhesive bonds might be stronger than the softer of the two bonded materials, and that the shear strength of the softer material, not the bond strength, should be used in friction models.

Traditional friction models, largely developed for metal-on-metal sliding, have added the force contribution due to the shear of junctions to the contribution from plowing, giving the extended expression:

\[
\mu = \frac{\tau A_r}{P} + \frac{(\tan \theta)}{\pi}
\]  

(2.19)

where \( A_r \) is the real area of contact and \( \tau \) is the shear strength of the material being plowed. This type of expression has met with relatively widespread acceptance in the academic community and is often used as the basis for other sliding friction models. But if the tip of the cone wears down, three contributions to the plowing process can be identified: the force needed to displace material from in front of the cone, the friction force along the leading face of the cone (i.e., the component in the macroscopic sliding direction), and the friction associated with shear of the interface along the worn frustum of the cone. From this analysis, it is clear that friction on two scales is involved: the macroscopic friction force for the entire
system, and the friction forces associated with the flow of material along the face of the cone and across its frustum. That situation is somewhat analogous to the interpretation of orthogonal cutting of metals in which the friction force of the chip moving up along the rake face of the tool and friction along the wear land are not in general the same as the cutting force for the tool as a whole [5]. Considering the three contributions to the friction of a flat-tipped cone gives

$$\mu = (\tau/P)\left(\pi r^2\right) + \mu_i \cos^2 \theta + \tan \theta / \pi$$  \hspace{1cm} (2.20)

where \( r \) is defined as the radius of the top of the worn cone and \( \mu_i \) is the friction coefficient of the cone against the material flowing across its face. Eq. 2.20 helps explain why the friction coefficients for ceramics and metals sliding on faceted diamond films are 10 or more times higher than the friction coefficients reported for smooth surfaces of the same materials sliding against smooth surfaces of diamond (i.e., \( \mu \gg \mu_i \)). When the rake angle \( \theta \) is small, \( \cos^2 \theta \) is close to 1.0, and the second term is only slightly less than \( \mu_i \) (0.02–0.12 typically). If one assumes that the friction coefficient for the material sliding across the frustum of the cone is the same as that for sliding along its face (\( \mu_i \)), then Eq. 2.20 can be re-written:

$$\mu = 2\mu_i + (\tan \theta) / \pi$$  \hspace{1cm} (2.21)

Thus, implying that the friction coefficient for a rigid sliding cone is more than twice that for sliding a flat surface of the same two materials. It is interesting to note that Eq. 2.21 does not account for the depth of penetration, a factor that seems critical for accounting for the energy required to plow through the surface (displace the volume of material ahead of the slider), and at \( \theta = 90^\circ \), which implies infinitely deep penetration of the cone, it would be impossible to move the slider at all as \( \mu \) tends toward infinity.

When one the complexities of surface finish it seems remarkable that Eqs. 2.20 and 2.21, which depend on a single quantity \( (\tan \theta) / \pi \), should be able to predict the friction coefficient with any degree of accuracy. The model is based on a single conical asperity cutting through a surface that makes no obvious accountability for multiple contacts and differences in contact angle. The model is also based on a surface’s relatively ductile response to a perfectly rigid asperity and can neither account for fracture during wear nor account for the change in the groove geometry that one would expect for multiple passes over the same surface.

Mulhearn and Samuels [47] published a paper on the transition between abrasive asperities cutting through a surface and plowing through it. The results of their experiments suggested that there exists a critical rake angle for that type of transition. (Note: The rake angle is the angle between the normal to the surface and the leading face of the asperity, with negative values indicating a tilt toward the direction of travel.) If plowing can occur only up to the critical rake angle, then we may compute the maximum contribution to friction due to plowing from the data of Mulhearn and Samuels and Eq. 2.18 (Table 2.6). This approach suggests that the maximum contribution of plowing to the friction coefficient of aluminium or nickel is about 0.03 in contrast to copper, whose maximum plowing contribution is 0.32.
Since the sliding friction coefficient for aluminium can be quite high (over 1.0 in some cases), the implication is that factors other than plowing, such as the shearing of strongly adhering junctions, would be the major contributor. Examination of unlubricated sliding wear surfaces of both Al and Cu often reveals a host of ductile-appearing features not in any way resembling cones, and despite the similar appearances in the microscope of worn Cu and Al, one finds from the first and last rows in Table 2.6 that the contribution of plowing to friction should be different by a factor of 10. Again, the simple cone model appears to be too simple to account for the difference.

Hokkirigawa and Kato [26] carried the analysis of abrasive contributions to sliding friction even further using observations of single hemispherical sliding contacts (quenched steel, tip radius 26 or 62 μm) on brass, carbon steel, and stainless steel in a scanning electron microscope. They identified three modes: (a) plowing (b) wedge formation and (c) cutting (chip formation). The tendency of the slider to produce the various modes was related to the degree of penetration, $D_p$. Here, $D_p = h/a$, where $h$ is the groove depth and $a$ is the radius of the sliding contact. The sliding friction coefficient was modeled in three ways depending upon the regime of sliding. Three parameters were introduced:

$$f = \frac{p}{\tau}, \quad \theta = \sin^{-1}(a/R)$$

and $\beta$, the angle of the stress discontinuity (shear zone) from Challen and Oxley’s [14] analysis. Where $p$ is the contact pressure, $\tau$ is the bulk shear stress of the flat specimen, and $R$ is the slider tip radius. The friction coefficient was given as follows for each mode:

**Cutting mode:**

$$\mu = \tan \left[ \theta - \left( \frac{\pi}{4} + \frac{1}{2} \cos^{-1} f \right) \right]$$  \hspace{1cm} (2.22)

**Wedge-forming mode:**

$$\mu = \frac{1 - \sin 2\beta + (1 - f^2)^{1/2}}{1 - \sin 2\beta + (1 - f^2)^{1/2}} \sin \theta + f \cos \theta$$

$$\mu = \frac{1}{1 - \sin 2\beta + (1 - f^2)^{1/2}} \cos \theta + f \sin \theta$$  \hspace{1cm} (2.23)

**Plowing mode:**

$$\mu = \frac{A \sin \theta + \cos(\cos^{-1} f - \theta)}{A \sin \theta + \cos(\cos^{-1} f - \theta)}$$  \hspace{1cm} (2.24)

Where

$$A = 1 + \left( \frac{\pi}{2} \right) + \cos^{-1} f - 2\theta - 2\sin^{-1} \frac{\sin \theta}{(1 - f)^{-1/2}}$$  \hspace{1cm} (2.25)

For unlubricated conditions, the transitions between the various modes were experimentally determined by observation in the scanning electron microscope.
Table 2.7 summarizes those results. Results of the study illustrate the point that the analytical form of the frictional dependence on the shape of asperities cannot ignore the mode of surface deformation. In summary, the foregoing treatments of the plowing contribution to friction assumed that asperities could be modeled as regular geometric shapes. However, rarely do such shapes appear on actual sliding surfaces. The asperities present on most sliding surfaces are irregular in shape, as viewed with a scanning electron microscope.

(c) Plowing with Debris Generation. Even when the predominant contribution to friction is initially from cutting and plowing of hard asperities through the surface, the generation of wear debris that submerges the asperities can reduce the severity of plowing. Table 2.8 shows that starting with multiple hard asperities of the same geometric characteristics produced different initial and steady-state friction coefficients for the three slider materials. Wear debris accumulation in the contact region affected the frictional behavior. In the case of abrasive papers and grinding wheels, this is called loading. Loading is extremely important in grinding, and a great deal of effort has been focused on dressing grinding wheels to improve their material removal efficiency. One measure of the need for grinding wheel dressing is an increase in the tangential grinding force or an increase in the power drawn by the grinding spindle.

As wear progresses, the wear debris accumulates between the asperities and alters the effectiveness of the cutting and plowing action by covering the active points. If the cone model is to be useful at all for other than pristine surfaces, the effective value of \( \theta \) must be given as a function of time or number of sliding passes. Not only is the wear rate affected, but the presence of debris affects the interfacial shear strength, as is explained later in this chapter in regard to third-body particle effects on friction. The observation that wear debris can accumulate and so affect friction has led investigators to try patterning surfaces to create pockets where debris can be collected [60]. The orientation and depths of the ridges and grooves in a surface affect the effectiveness of the debris-trapping mechanism.

(d) Plowing with Adhesion. Traditional models for sliding friction have historically been developed with metallic materials in mind. Classically, the friction force is said to be an additive contribution of adhesive (\( S \)) and plowing forces (\( F_{pl} \)) [8]:

\[
F = S + F_{pl}
\] (2.26)
The adhesive force derives from the shear strength of adhesive metallic junctions that are created when surfaces touch one another under a normal force. Thus, by dividing by the normal force we find that

$$l = \frac{l_{adhesion}}{l_{plowing}}.$$ 

If the shear strength of the junction is $s$ and the contact area is $A$, then

$$S = sA.$$ 

The plowing force $F_{pl}$ is given by

$$F_{pl} = pA'.$$ 

Where $p$ is the mean pressure to displace the metal in the surface and $A'$ is the cross section of the grooved wear track. While helpful in understanding the results of experiments in the sliding friction of metals, the approach involves several applicability-limiting assumptions, for example, that adhesion between the surfaces results in bonds that are continually forming and breaking, that the protuberances of the harder of the two contacting surfaces remain perfectly rigid as they plow through the softer counterpart, and perhaps most limiting of all, that the friction coefficient for a tribosystem is determined only from the shear strength properties of materials.

(e) Single-Layer Shear (SLS) Models. SLS models for friction depict an interface as a layer whose shear strength determines the friction force, and hence, the friction coefficient. The layer can be a separate film, like a solid lubricant, or simply the near surface zone of the softer material that is shearing during friction. The friction force $F$ is the product of the contact area $A$ and the shear strength of the layer:

$$S = TA.$$ 

The concept that the friction force is linearly related to the shear strength of the interfacial material has a number of useful implications, especially as regards the role of thin lubricating layers, including oxides and tarnish films. It is known from the work of Bridgman [9] on the effects of pressure on mechanical properties that $\tau$ is affected by contact pressure, $p$:

$$\tau = \tau_0 + \alpha p.$$ 

<table>
<thead>
<tr>
<th>Slider material</th>
<th>24 µm Grit size</th>
<th>16 µm Grit size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Starting $\mu$</td>
<td>Ending $\mu$</td>
</tr>
<tr>
<td>AISI 52100 steel</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>2014-T4 aluminium</td>
<td>0.69</td>
<td>0.56</td>
</tr>
<tr>
<td>PMMA</td>
<td>0.73</td>
<td>0.64</td>
</tr>
</tbody>
</table>

* Normal force 2.49 N, sliding speed 5 mm/sec, multiple strokes 20 mm long
Table 2.9 lists several values for the shear stress and the constant $\alpha$ (Kragelskii et alia [36]).

(f) Multiple-Layer Shear (MLS) Models. SLS models presume that the sliding friction can be explained on the basis of the shear strength on a single layer interposed between solid surfaces. Evidence revealed by the examination of frictional surfaces suggests that shear can occur at various positions in the interface: for example, at the upper interface between the solid and the debris layer, within the entrapped debris or transfer layer itself, at the lower interface, or even below the original surfaces where extended delaminations may occur. Therefore, one may construct a picture of sliding friction that involves a series of shear layers (sliding resistances) in parallel. Certainly, one would expect the predominant frictional contribution to be the lowest shear strength in the shear layers. Yet the shear forces transmitted across the weakest interface may still be sufficient to permit some displacement to occur at one or more of the other layers above or below it, particularly if the difference in shear strengths between those layers is small.

The MLS models can be treated like electrical resistances in a series. The overall resistance of such a circuit is less than any of the individual resistances because multiple current paths exist. Consider, for example, the case where there are three possible operable shear planes stacked up parallel to the sliding direction in the interface. Then,

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}$$  \hspace{1cm} (2.31)

And, solving for the total friction force $F$, in terms of the friction forces acting on the three layers, is

$$F = \frac{F_1 F_2 F_3}{F_1 F_2 + F_2 F_3 + F_1 F_3}$$  \hspace{1cm} (2.32)

If the area of contact $A$ is the same across each layer, then Eq. 2.32 can be written in terms of the friction coefficient of the interface, the shear stresses of each layer, and the normal load $P$ as follows:

Table 2.9 Measured values for the shear stress dependence on pressure [31](Reproduced with permission)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\tau_0$ (kgf/mm$^2$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>3.00</td>
<td>0.043</td>
</tr>
<tr>
<td>Beryllium</td>
<td>0.45</td>
<td>0.250</td>
</tr>
<tr>
<td>Chromium</td>
<td>5.00</td>
<td>0.240</td>
</tr>
<tr>
<td>Copper</td>
<td>1.00</td>
<td>0.110</td>
</tr>
<tr>
<td>Lead</td>
<td>0.90</td>
<td>0.014</td>
</tr>
<tr>
<td>Platinum</td>
<td>9.50</td>
<td>0.100</td>
</tr>
<tr>
<td>Silver</td>
<td>6.50</td>
<td>0.090</td>
</tr>
<tr>
<td>Tin</td>
<td>1.25</td>
<td>0.012</td>
</tr>
<tr>
<td>Vanadium</td>
<td>1.80</td>
<td>0.250</td>
</tr>
<tr>
<td>Zinc</td>
<td>8.00</td>
<td>0.020</td>
</tr>
</tbody>
</table>
\[ \mu = \left( \frac{A}{P} \right) \left[ \frac{\tau_1 \tau_2 \tau_3}{\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_1 \tau_3} \right] \] (2.33)

If one of the shear planes suddenly became unable to deform (say, by work hardening or by clogging with a compressed clump of wear debris), the location of the governing plane of shear may shift quickly, causing the friction to fluctuate. Thus, by writing the shear stresses of each layer as functions of time, the MLS model has the advantage of being able to account for variations in friction force with time and may account for some of the features observed in microscopic examinations of wear tracks.

(g) Molecular Dynamics’ Models. When coupled with information from nanoprobe instruments, such as the atomic force microscope, the scanning tunneling microscope, the surface-forces apparatus, and the lateral-force microscope, MD studies have made possible insights into the behavior of pristine surfaces on the atomic scale. Molecular dynamics models of friction for assemblages of even a few hundred atoms tend to require millions upon millions of individual, iterative computations to predict frictional interactions taking place over only a fraction of a second in real time, because they begin with very specific arrangements of atoms, usually in single crystal form with a specific sliding orientation, results are often periodic with sliding distance. Some of the calculation results are remarkably similar to certain types of behavior observed in real materials, simulating such phenomena as dislocations (localized slip on preferred planes) and the adhesive transfer of material to the opposing counterface. However, molecular dynamics models are not presently capable of handling such contact surface features as surface fatigue-induced delaminations, wear debris particles compacting and deforming in the interface, high-strain-rate phenomena, work hardening of near-surface layers, and effects of inclusions and other artifacts present in the microstructures of commercial engineering materials.

The models presented up to this point use either interfacial geometric parameters or materials properties (i.e., bonding energies, shear strengths, or other mechanical properties) to predict friction. Clearly, frictional heating and the chemical environment may affect some of the variables used in these models. For example, the shear strength of many metals decreases as the temperature increases and increases as the speed of deformation increases. Certainly, wear and its consequences (debris) will affect friction. Thus, any of the previously described models will probably require some sort of modification, depending on the actual conditions of sliding contact. In general, the following can be said about friction models:

1. No existing friction model explicitly accounts for all the possible factors that can affect friction;
2. Even very simple friction models may work to some degree under well-defined, limited ranges of conditions, but their applicability must be tested in specific cases;
3. Accurately predictive, comprehensive tribosystem-level models that account for interface geometry, materials properties, lubrication aspects, thermal, chemical, and external mechanical system response, all in a time-dependent context, do not exist;
4. Friction models should be selected and used based on an understanding of their limitations and on as complete as possible an understanding of the dominant influences in the tribosystem to which the models will be applied; and
5. Current quantitative models produce a single value for the friction force, or friction coefficient. Since the friction force in nearly all known tribosystems varies to some degree, any model that predicts a single value is questionable.

If no existing model is deemed appropriate, the investigator could either modify a current model to account for the additional variables, develop a new system-specific model, or revert to simulative testing and/or field experiments to obtain the approximate value. An alternative to modeling is to estimate frictional behavior using a graphical, or statistical approach.

2.2.3 Frictional Heating

Heat generation and rising surface temperatures are intuitively associated with friction. When a friction force $F$ moves through a distance $x$, an amount of energy $Fx$ is produced. The laws of thermodynamics require that the energy so produced be dissipated to the surroundings. At equilibrium, the energy into a system $U_{in}$ equals the sum of the energy output to the surroundings $U_{out}$ (dissipated externally) and the energy accumulated $U_{accumulated}$ (consumed or stored internally):

$$U_{in} = U_{out} + U_{accumulated} \quad (2.34)$$

The rate of energy input in friction is the product of $F$ and the sliding velocity $v$ whose units work out to energy per unit time (e.g., Nm/sec). This energy input rate at the frictional interface is balanced almost completely by heat conduction away from the interface, either into the contacting solids or by radiation or convection to the surroundings. In general, only a small amount of frictional energy, perhaps only 5%, is consumed or stored in the material as microstructural defects such as dislocations, the energy to produce phase transformations, surface energy of new wear particles and propagating subsurface cracks, etc. Most of the frictional energy is dissipated as heat. Under certain conditions, there is enough heat to melt the sliding interface. Energy that cannot readily be conducted away from the interface raises the temperature locally. Assuming that the proportionality of friction force $F$ to normal force $P$ (i.e., by definition, $F = \mu P$) holds over a range of normal forces, we would expect that the temperature rise in a constant-velocity sliding system should increase linearly with the normal force. Tribologists distinguish between two temperatures, the flash temperature and the mean surface temperature. The former is localized, the latter averaged out over the nominal contact zone. Since sliding surfaces touch at only a few locations at any instant, the energy is concentrated there and the heating is particularly intense—thus, the name flash temperature. The combined effect of many such flashes dissipating their energy in the interface under steady state is to heat a near-surface layer to an average
temperature that is determined by the energy transport conditions embodied in Eq. 5.34 given earlier. Blok [6] discussed the concept and calculation of flash temperature in a review article. The early work of Blok [7] and Jaeger [32] is still cited as a basis for more recent work, and it has been reviewed in a simplified form by Bowden and Tabor [8]. Basically, the temperature rise in the interface is given as a function of the total heat developed, \( Q \):

\[
Q = \frac{\mu W g v}{J}
\]

where \( \mu \) is the sliding friction coefficient, \( W \) is the load, \( g \) the acceleration due to gravity, \( v \) the sliding velocity, and \( J \) the mechanical equivalent of heat (4.186 J/cal). Expressions for various heat flow conditions are then developed based on Eq. 2.35. Some of these are given in Table 2.10.

As Table 2.10 shows, the expressions become more complicated when the cooling effects of the incoming, cooler surface are accounted for. Rabinowicz [50] published an expression for estimating the flash temperature rise in sliding:

\[
\theta_m = \frac{v}{2} (\pm \text{a factor of 2 to 3})
\]

Where, \( v \) is sliding velocity (ft/min) and \( \theta_m \) is the estimated surface flash temperature (°F). A comparison of the results of using Eq. 2.36 with several other, more complicated models for frictional heating has provided similar results, but more rigorous treatments are sometimes required to account for the variables left out of this rule of thumb. In general, nearly all models for flash or mean temperature rise during sliding contain the friction force–velocity product. Sometimes, the friction force is written as the product of the normal force and friction coefficients.

A review of frictional heating calculations has been provided by Cowan and Winer [15], along with representative materials properties data to be used in those calculations. Their approach involves the use of two heat partition coefficients (\( \gamma_1 \) and \( \gamma_2 \)) that describe the relative fractions of the total heat that go into each of the contacting bodies, such that \( \gamma_1 + \gamma_2 = 1 \). The time that a surface is exposed to frictional heating will obviously affect the amount of heat it receives. The Fourier

---

**Table 2.10** Temperature rise during sliding, after Jaeger [32, 31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Temperature Rise (^b) ((T = T_o))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular junction of radius ( a )</td>
<td>( \frac{Q}{4a(k_1+k_2)} )</td>
</tr>
<tr>
<td>Square junction of side = 2 ( l ), at low speed</td>
<td>( \frac{Q}{24l(k_1+k_2)} )</td>
</tr>
<tr>
<td>Square junction of side = 2 ( l ), at high speed wherein the slider is being cooled by the incoming surface of the flat disk specimen</td>
<td>( \frac{Q^{1/2}}{3.76[k_1(\rho v)^{1/2}+1.125v^{1/2}k_2]} )</td>
</tr>
</tbody>
</table>

Where \( x = \left( k_1/\rho c_1 \right) \) for the disk material

\( T \) = steady-state junction temperature, \( T_o \) = initial temperature, \( k_{1,2} \) = thermal conductivity of the slider and flat bodies, \( \rho \) = density, \( c \) = specific heat
modulus, $F_o$, a dimensionless parameter, is introduced to establish whether or not steady-state conditions have been reached at each surface. For a contact radius $a$, an exposure time $t$, and a thermal diffusivity for body $i$ of $D_i$,

$$F_o = \frac{D_i t}{a^2}$$

(2.37)

The Fourier modulus is taken to be 100 for a surface at steady state conditions. Another useful parameter grouping is the Peclet number $P_e$, defined in terms of the density of the solid $\rho$, the specific heat $c_p$, the sliding velocity $v$, the thermal conductivity $k$, and the characteristic length $L_c$:

$$P_e = \frac{\rho c_p v L_c}{k}$$

(2.38)

The characteristic length is the contact width for a line contact or the contact radius for a circular contact. The Peclet number relates the thermal energy removed by the surrounding medium to that conducted away from the region in which frictional energy is being dissipated. As $D_i = (\rho c_p/k)$ yields the following,

$$P_e = \frac{v L_c}{D_i}$$

(2.39)

The Peclet number is sometimes used as a criterion for determining when to apply various forms of frictional heating models. Peclet number is used in understanding frictional heating problems associated with grinding and machining processes. It is important to compare the forms of models derived by different authors for calculating flash temperature rise. Four treatments for a pin moving along a stationary flat specimen are briefly compared: Rabinowicz’s derivation based on surface energy considerations, a single case from Cowan and Winer’s review, Kuhlmann-Wilsdorf’s model, and the model provided by Ashby. Based on considerations of junctions of radius $r$ and surface energy of the softer material $\Gamma$, Rabinowicz arrived at the following expression:

$$T_f = \frac{3000 \pi \mu \Gamma v}{J (k_1 + k_2)}$$

(2.40)

where $J$ is the mechanical equivalent of heat, $v$ is sliding velocity, $\mu$ is the friction coefficient, and $k_1$ and $k_2$ represent the thermal conductivities of the two bodies. The constant 3000 obtained from the calculation of the effective contact radius $r$ in terms of the surface energy of the circular junctions $\Gamma$ and their hardness $H$ (i.e., $r = 12,000 \Gamma H$) and the load carried by each asperity ($P = \pi r^2 H$). Thus, the numerator is actually the equivalent of $F v$ expressed in terms of the surface energy model. The equation provided by Cowan and Winer, for the case of a circular contact with one body in motion is

$$T_f = \frac{\gamma_1 \mu P v}{\pi a k_1}$$

(2.41)
where \( c_1 \) is the heat partition coefficient, described earlier, \( P \) is the normal force, \( a \) is the radius of contact, and \( k_1 \) is as defined earlier. The value of \( c_1 \) takes various forms depending on the specific case. The presence of elastic, or plastic contact, can also affect the form of the average flash temperature, as Table 2.11 demonstrates. Here, the exponents of normal force and velocity are not unity in all cases. Kuhlmann-Wilsdorf [38] considered an elliptical contact area as the planar moving heat source. The flash temperature is given in terms of the average temperature in the interface \( T_{\text{ave}} \):

\[
T_{\text{ave}} = \frac{\pi qr}{4k_1}
\] (2.42)

Where \( q \) is the rate of heat input per unit area (related to the product of friction force and velocity), \( r \) is the contact spot radius, and \( k_1 \) is the thermal conductivity, as given earlier. Then

\[
T_f = \frac{T_{\text{ave}}}{\left(1/ZS\right) + \left(k_1/S_O\right)}
\] (2.43)

where \( Z \) is a velocity function and \( S \) and \( S_O \) are contact area shape functions (both = 1.0 for circular contact). At low speeds, where the relative velocity of the surfaces \( v_r < 2(v_r = v/P_e) \), \( Z \) can be approximated by \( 1/[1 + (v_r/3)] \). The differences between models for frictional heating arise from the following:

1. Assuming different shapes for the heat source on the surface;
2. Different ways to partition the flow (dissipation) of heat between sliding bodies;
3. Different ways to account for thermal properties of materials (e.g., using thermal diffusivity instead of thermal conductivity, etc.);
4. Different contact geometry (sphere-on-plane, flat-on-flat, cylinder-on-flat, etc.);
5. Assuming heat is produced from a layer (volume) instead of a planar area; and
6. Changes in the form of the expression as the sliding velocity increases.

Comparing the temperature rises predicted by different models for low sliding speeds produces accurate results, even with the uncertainties in the values of the
material properties that go into the calculations. At higher speeds, the predictions become unreliable since materials properties change as a function of temperature and the likelihood of the interface reaching a steady state is much lower. Experimental studies have provided very useful information in validating the forms of frictional heating models. Experimental scientists have often used embedded thermocouples in one or both members of the sliding contact to measure surface temperatures, and others sometimes made thermocouples out of the contacts themselves. However, techniques using infrared sensors have been used as well. Dow and Stockwell [16] used infrared detectors with a thin, transparent sapphire blade sliding on a 15-cm-diameter ground cylindrical drum to study the movements and temperatures of hot spots. Griffioen et al. [22] and Quinn and Winer [49] used an infrared technique with a sphere-on-transparent sapphire disk geometry. A similar arrangement was also developed and used by Furey with copper, iron, and silver spheres sliding on sapphire, and Enthoven et al. [18]. used an infrared system with a ball-on-flat arrangement to study the relationship between scuffing and the critical temperature for its onset.

Frictional heating is important because it changes the shear strengths of the materials in the sliding contact, promotes reactions of the sliding surfaces with chemical species in the environment, enhances diffusion of species, and can result in the breakdown or failure of the lubricant to perform its functions. Under extreme conditions, such as plastic extrusion, frictional heating can result in molten layer formation that serves as a liquid lubricant.

2.3 Lubrication to Control Friction in Machining (after Jackson and Morrell [31])

The frictional characteristics of liquid and solid lubricants and their interaction with materials are reviewed. Comprehensive discussions of the mechanical and chemical engineering aspects of lubrication are available in the literature [62]. The following section was originally published by Jackson and Morrell [31] and appears hereafter.

2.3.1 Liquid Lubrication

The process of lubrication is one of supporting the contact pressure between opposing surfaces, helping to separate them, and at the same time reducing the sliding or rolling resistance in the interface. There are several ways to accomplish this. One way is to create in the gap between the bodies geometric conditions that produce a fluid pressure sufficient to prevent the opposing asperities from touching while still permitting shear to be fully accommodated within the fluid. That
method relies on fluid mechanics and modifications of the lubricant chemistry to tailor the liquid’s properties. Another way to create favorable lubrication conditions is to formulate the liquid lubricant in such a way that chemical species within it react with the surface of the bodies to form shearable solid films. Surface species need not react with the lubricant, but catalyze the reactions that produce these protective films.

Several attributes of liquids make them either suitable or unsuitable as lubricants. Klaus and Tewksbury [33] have discussed these characteristics in some detail. They include:
1. Density;
2. Bulk modulus;
3. Gas solubility;
4. Foaming and air entrainment tendencies;
5. Viscosity and its relationships to temperature and pressure;
6. Vapor pressure;
7. Thermal properties and stability; and
8. Oxidation stability.

The viscosity of fluids usually decreases with temperature and therefore can reduce the usefulness of a lubricant as temperature rises. The term viscosity index, abbreviated VI, is a means to express this variation. The higher the VI, the less the change in viscosity with temperature. One of the types of additives used to reduce the sensitivity of lubricant viscosity to temperature changes is called a VI improver. ASTM test method D 2270 is one procedure used to calculate the VI. The process is described step-by-step in the article by Klaus and Tewksbury [33]. The method involves references to two test oils, the use of two different methods of calculation (depending on the magnitude of VI), and relies on charts and tables.

ASTM Standard D341 recommends using the Walther equation to represent the dependence of lubricant viscosity on temperature. Defining \( Z \) as the viscosity in cSt plus a constant (typically ranging from 0.6 to 0.8 with ASTM specifying 0.7), \( T \) equal to the temperature in Kelvin or Rankin, and \( A \) and \( B \) being constants for a given oil, then

\[
\log_{10}(\log_{10} Z) = A + B(\log_{10} T) \quad (2.44)
\]

Sanchez-Rubio et al. [57] have suggested an alternative method in which the Walther equation is used. In this case, they define a viscosity number (VN) as follows:

\[
VN = \left[ 1 + \frac{(3.55 + B)}{3.55} \right] \times 100 \quad (2.45)
\]

The value of 3.55 was selected because lubricating oils with a VI of 100 have a value of \( B \) about equal to \(-3.55\). Using this expression implies that VN = 200 would correspond to an idealized oil whose viscosity has no dependence of viscosity on temperature (i.e., \( B = 0 \)). The pressure to which an oil is subjected to
can influence its viscosity. The relationship between dynamic viscosity and hydrostatic pressure $p$ can be represented by

$$\eta = \eta_o \exp(\alpha p)$$

(2.46)

where $\eta$ and $\alpha$ vary with the type of oil. Table 5.12 illustrates the wide range of viscosities possible for several liquid lubricants under various temperatures and pressures. The viscosity indices for these oils range from $-132$ to $195$. Viscosity has a large effect on determining the regime of lubrication and the resultant friction coefficient. Similarly to the effect of strain rate on the shear strength of certain metals, like aluminium, the rate of shear in the fluid can also alter the viscosity of a lubricant.

Ramesh and Clifton [53] constructed a plate impact device to study the shear strength of lubricants at strain rates as high as 900,000/sec and found significant effects of shear rate on the critical shear stress of lubricants. In a Newtonian fluid, the ratio of shear stress to shear strain does not vary with stress, but there are other cases, such as for greases and solid dispersions in liquids, where the viscosity varies with the rate of shear. Such fluids are termed non-Newtonian and the standard methods for measuring viscosity cannot be used.

Lubrication regimes determine the effectiveness of fluid film formation, and hence, surface separation. In the first decade of the twentieth century, Stribeck developed a systematic method to understand and depict regimes of journal bearing lubrication, linking the properties of lubricant viscosity ($\eta$), rotational velocity of a journal ($\omega$), and contact pressure ($p$) with the coefficient of friction. Based on the work of Mersey, McKee, and others, the dimension-less group of parameters has evolved into the more recent notation ($ZN/p$), where $Z$ is viscosity, $N$ is rotational speed, and $p$ is pressure. The Stribeck curve has been widely used in the design of bearings and to explain various types of behavior in the field of lubrication. At high pressures, or when the lubricant viscosity and/or speed are very low, surfaces may touch, leading to high friction. In that case, friction coefficients are typically in the range of 0.5–2.0. The level plateau at the left of the curve represents the boundary lubrication regime in which friction is lower than for unlubricated sliding contact ($\mu = 0.05$ to about 0.15). The drop-off in friction is called the mixed film regime. The mixed regime refers to a combination of boundary lubrication with hydrodynamic or elastohydrodynamic lubrication.

### Table 2.12 Effects of temperature and pressure on viscosity of selected lubricants having various viscosity indexes (All fluids have viscosities of 20 °C St at 40 °C and 0.1 MPa Pressure) [31] (Reproduced with permission)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Fluorolube</th>
<th>Hydrocarbon</th>
<th>Ester</th>
<th>Silicone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity index</td>
<td>$-132$</td>
<td>100</td>
<td>151</td>
<td>195</td>
</tr>
<tr>
<td>Viscosity (cSt) at $-40 ^\circ$C</td>
<td>500,000</td>
<td>14,000</td>
<td>3,600</td>
<td>150</td>
</tr>
<tr>
<td>Viscosity (cSt) at $-100 ^\circ$C</td>
<td>2.9</td>
<td>3.9</td>
<td>4.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Viscosity (cSt) at $-40 ^\circ$C and 138 MPa</td>
<td>2700</td>
<td>340</td>
<td>110</td>
<td>160</td>
</tr>
<tr>
<td>Viscosity (cSt) at $-40 ^\circ$C and 552 MPa</td>
<td>$&gt;1,000,000$</td>
<td>270,000</td>
<td>4,900</td>
<td>48,000</td>
</tr>
</tbody>
</table>
Beyond the minimum in the curve, hydrodynamic and elastohydrodynamic lubrication regimes are said to occur. Friction coefficients under such conditions can be very low. Typical friction coefficients for various types of rolling element bearings range between 0.001 and 0.0018.

The conditions under which a journal bearing of length $L$, diameter $D$, and radial clearance $C$ (bore radius minus bearing shaft radius) operates in the hydrodynamic regime can be summarized using a dimensionless parameter known as the Sommerfeld number $S$, defined by

$$ S = \frac{\eta N L D}{P} \left( \frac{R}{C} \right)^2 $$

(2.47)

where $P$ is the load on the bearing perpendicular to the axis of rotation, $N$ is the rotational speed, $\eta$ is the dynamic viscosity of the lubricant, and $R$ is the radius of the bore. The more concentrically the bearing operates, the higher the value of $S$, but as $S$ approaches 0, the lubrication may fail, leading to high friction. Sometimes Strubeck curves are plotted using $S$ instead of $(ZN/p)$ as the abscissa. Raimondi and his co-workers [52] added leakage considerations when they developed design charts in which the logarithm of the Sommerfeld number is plotted against the logarithm of either the friction coefficient or the dimensionless film thickness.

Using small journal bearings, McKee developed the following expression for the coefficient of friction $\mu$ based on the journal diameter $D$, the diametral clearance $C$, and an experimental variable $k$, which varies with the length to diameter ratio $(L/D)$ of the bearing (Hall et al. [24]):

$$ \mu = \left(4.73 \times 10^{-8}\right) \left(\frac{ZN}{p}\right) \left(\frac{D}{C}\right) + k $$

(2.48)

The value of $k$ is about 0.015 at $(L/D) = 0.2$, drops rapidly to a minimum of about 0.0013 at $(L/D) = 1.0$, and rises nearly linearly to about 0.0035 at $(L/D) = 3.0$. A simpler expression, discussed by Hutchings [28], can be used for bearings that have no significant eccentricity:

$$ \mu = \frac{2\pi h}{S R} $$

(2.49)

where $S$ is the Sommerfeld number, $h$ is the mean film thickness, and $R$ is the journal radius. With good hydrodynamic lubrication and good bearing design, $\mu$ can be as low as 0.001.

Hydrodynamic lubrication, sometimes called thick-film lubrication, generally depends on the development of a converging wedge of lubricant in the inlet of the interface. This wedge generates a pressure profile to force the surfaces apart. When the elastic deformation of the solid bodies is similar in extent to the thickness of the lubricant film, then elastohydrodynamic lubrication is said to occur. This latter regime is common in rolling element bearings and gears where high Hertz contact stresses occur. If the contact pressure exceeds the elastic limit of the surfaces,
plastic deformation and increasing friction occur. One way to understand and control the various lubrication regimes is by using the specific film thickness (also called the lambda ratio), defined as the ratio of the minimum film thickness in the interface \( h \) to the composite root-mean-square (rms) surface roughness \( \sigma^* \):

\[
\Lambda = h/\sigma^* \tag{2.50}
\]

where the composite surface roughness is defined in terms of the rms roughness \( \sigma_{1,2} \) of surfaces 1 and 2, respectively:

\[
\sigma^* = \sqrt{\left( \sigma_1^2 + \sigma_2^2 \right)} \tag{2.51}
\]

For the boundary regime, \( \Lambda \ll 1 \). For the mixed regime \( 1 < \Lambda < 3 \). For the hydrodynamic regime, \( \Lambda \gg 6 \), and for the elastohydrodynamic regime, \( 3 < \Lambda < 10 \). Boundary lubrication produces friction coefficients that are lower than those for unlubricated sliding but higher than those for effective hydrodynamic lubrication, typically in the range \( 0.05 < \mu < 0.2 \). Briscoe and Stolarski [10] have reviewed friction under boundary-lubricated conditions. They cited the earlier work of Bowden, which gave the following expression for the friction coefficient under conditions of boundary lubrication:

\[
\mu = \beta \mu_a + (1 - \beta)\mu_1 \tag{2.52}
\]

where the adhesive component \( \mu_a \) and the viscous component of friction \( \mu_1 \) are given in terms of the shear stress of the adhesive junctions in the solid (metal) \( \tau_m \) and the shear strength of the boundary film \( \tau_1 \) under the influence of a contact pressure \( \sigma_p \):

\[
\mu_a = \frac{\tau_m}{\sigma_p} \tag{2.53}
\]

The parameter, \( \beta \), is called the fractional film defect [10] and is:

\[
\beta = 1 - \exp\left\{-\left[\frac{(30.9 \times 10^5)T_m^{1/2}}{VM^{1/2}}\right]\exp\left(-\frac{E_c}{RT}\right)\right\} \tag{2.54}
\]

where \( M \) is the molecular weight of the lubricant, \( V \) is the sliding velocity, \( T_m \) is the melting temperature of the lubricant, \( E_c \) is the energy to desorb the lubricant molecules, \( R \) is the universal gas constant, and \( T \) is the absolute temperature. Various graphical methods have been developed to help select boundary lubricants and to help simplify the task of bearing designers. Most of these methods are based on the design parameters of bearing stress (or normal load) and velocity. One method, developed by Glaeser and Dufrane [23] involves the use of design charts for different bearing materials. An alternate but similar approach was used in developing the so-called IRG transitions diagrams (subsequently abbreviated ITDs), an approach that evolved in the early 1980s, was applied to various bearing steels, and is still being used to define the conditions under which boundary-lubricated tribosystems operate.
effectively. Instead of pressure, load is plotted on the ordinate. Three regions of ITDs are defined in terms of their frictional behavior: Region I, in which the friction trace is relatively low and smooth, Region II, in which the friction trace begins with a high level then settles down to a lower, smoother level, and Region III, in which the friction trace is irregular and remains high. The transitions between Regions I and II or between Regions I and III are described as a collapse of liquid film lubrication. The locations of these transition boundaries for steels were seen to depend more on the surface roughness of the materials and the composition of the lubricants and less on microstructure and composition of the alloys. Any of the following testing geometries can be used to develop ITDs: four-ball machines, ball-on-cylinder machines, crossed-cylinders machines, and flat-on-flat testing machines (including flat-ended pin-on-disk). One important aspect of the use of liquid lubricants is how they are applied, filtered, circulated, and replenished. Lubricants can also be formed on surfaces by the chemical reaction of vapor-phase precursor species in argon and nitrogen environments.


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