This book is meant to serve as a text for students and a reference for professionals on the basic theory and methods used for stochastic modeling and analysis of marine structures subjected to environmental loads. The first part of the book provides a detailed introduction to the basic dynamic analysis of structures, which serves as a foundation for later chapters on stochastic response analysis. This includes an extensive chapter on the finite element method. A careful introduction to stochastic modeling is provided, which includes the concepts of stochastic process, variance spectrum, random environmental processes, response spectrum, response statistics, and short- and long-term extreme value models. The second part of the book offers detailed discussions of limit state design approaches, fatigue design methods, equations of motion for dynamic structures, and numerical solution techniques. The final chapter highlights methods for prediction of extreme values from measured data or data obtained by Monte Carlo simulation.

Arvid Naess has been a professor of structural engineering since 1987 and a professor of mathematical statistics since 2001 at the Norwegian University of Science and Technology. He works on a wide range of problems related to stochastic dynamics of structures and structural safety and reliability. Professor Naess has published more than 250 scientific papers and lectured at conferences and universities worldwide. He is the associate editor of several international engineering journals. Professor Naess is a recipient of the Alfred M. Freudenthal Medal from ASCE and is an elected Fellow of the ASME and ASCE.

Torgeir Moan has been a professor of marine structures at the Norwegian University of Science and Technology since 1977. He was the first (adjunct) Keppel professor at the National University of Singapore (2002–2007) and Director of the Centre for Ships and Ocean Structures (CeSOS), a Norwegian Centre of Excellence since 2002. Professor Moan’s work focuses on structural analysis and design of marine structures, with an emphasis on structural risk and reliability analysis, as well as probabilistic analysis of wave- and wind-induced stochastic dynamic load effects. He has published more than 450 refereed journal and conference papers. He is an editor of the Journal of Marine Structures and serves on the boards of several other journals. Professor Moan received various awards for his research, including the Statoil and the ASME James W. Rice Awards. He is an elected Foreign Member of the Royal Academy of Engineering in the UK and Fellow of ASCE, IABSE, and The Offshore Energy Center Hall of Fame.
Stochastic Dynamics of Marine Structures

Arvid Naess
Norwegian University of Science and Technology

Torgeir Moan
Norwegian University of Science and Technology
# Contents

## Preface

**Preface**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Preliminaries</td>
<td>xiii</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Equations of Motion</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Stochastic Models</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Organization of the Book</td>
<td>5</td>
</tr>
<tr>
<td>2 Dynamics of Single-Degree-of-Freedom Linear Systems</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Harmonic Oscillator – Free Vibrations</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Motions of Marine Structures</td>
<td>6</td>
</tr>
<tr>
<td>2.2.2 Translational Oscillations</td>
<td>7</td>
</tr>
<tr>
<td>2.2.3 Example – Amplitude and Phase of a Free Oscillation</td>
<td>9</td>
</tr>
<tr>
<td>2.2.4 Example – Heave Oscillations of a Spar Buoy</td>
<td>10</td>
</tr>
<tr>
<td>2.2.5 Example – Heave and Surge Oscilliations of a TLP</td>
<td>11</td>
</tr>
<tr>
<td>2.2.6 Rotational Oscillations</td>
<td>13</td>
</tr>
<tr>
<td>2.2.7 Example – Ideal Pendulum</td>
<td>14</td>
</tr>
<tr>
<td>2.2.8 Example – Tilting Oscillations of an ALP</td>
<td>14</td>
</tr>
<tr>
<td>2.2.9 Example – Pitch and Roll Oscilliations of a Semisubmersible</td>
<td>16</td>
</tr>
<tr>
<td>2.2.10 Example – Yaw Oscillations of a TLP</td>
<td>17</td>
</tr>
<tr>
<td>2.3 Free Damped Oscillations</td>
<td>17</td>
</tr>
<tr>
<td>2.3.1 Example – Critical Damping</td>
<td>19</td>
</tr>
<tr>
<td>2.3.2 Example – Logarithmic Decrement</td>
<td>22</td>
</tr>
<tr>
<td>2.3.3 Example – Vibrating Tower</td>
<td>23</td>
</tr>
<tr>
<td>2.3.4 Example – Coulomb Damping</td>
<td>27</td>
</tr>
<tr>
<td>2.4 Forced Vibrations by Harmonic Excitation</td>
<td>30</td>
</tr>
<tr>
<td>2.4.1 Example – Harmonic Force</td>
<td>34</td>
</tr>
<tr>
<td>2.4.2 Example – Damping Ratio from Half-Value Width</td>
<td>37</td>
</tr>
<tr>
<td>2.5 Forced Vibration – Complex Analysis</td>
<td>38</td>
</tr>
<tr>
<td>2.5.1 Example – Transfer Function</td>
<td>39</td>
</tr>
</tbody>
</table>
# Contents

2.5.2 Example – Structure on a Vibrating Foundation 40  
2.5.3 Example – Vibrating Beam 41  
2.6 Forced Vibrations by Periodic Excitation 42  
2.6.1 Example – Periodic Excitation 44  
2.7 Forced Vibrations by Arbitrary Excitation 46  
2.8 Impulse Response Function and Duhamel Integral 47  
2.8.1 Example – Suddenly Applied Force 51  
2.9 Maximum Response to Various Force Time Histories 51  
2.9.1 Example – Torsional Rotation of a Suspension Bridge 52  
2.9.2 Example – Response to Collision Load 55  

3 Dynamics of Multi-Degree-of-Freedom Linear Systems 57  
3.1 Introduction 57  
3.2 Discrete Systems 59  
3.2.1 Discrete Systems of Rigid Bodies 59  
3.2.2 Other Examples 60  
3.2.3 Vibrating Bars and Strings 63  
3.3 Beams Under Axial and Lateral Loads 66  
3.3.1 Basic Principles of Structural Mechanics 66  
3.3.2 Differential Equation for Dynamic Behavior 76  
3.3.3 Approximate Solution of Dynamic Response Based on Discretization 81  
3.3.4 Example – Simple Estimates of Lowest Eigenfrequency of Complex Structures 82  
3.3.5 Example – Cantilever Beam 86  
3.3.6 Example – Wind Turbines 88  
3.3.7 The Rayleigh-Ritz Method for Determining Mode Shapes and Natural Frequencies 90  
3.3.8 Example – Guyed Tower 92  
3.3.9 Example – Ship Vibration 93  
3.3.10 Modal Superposition 93  
3.3.11 Discussion of Forced Vibration of a Slender Beam 95  
3.3.12 Formulations for Moving Loads 97  

4 Finite Element Method 98  
4.1 Introduction 98  
4.2 Discretization 99  
4.3 Element Stiffness Relationship for a Bar Element 100  
4.3.1 Matrix Method 100  
4.3.2 Finite Element Method Based on Virtual Work (Galerkin’s Method) and Assumed Displacement 101  
4.3.3 Further Considerations on the Assumed Displacement for the Truss Element 103  
4.3.4 Load Vector 104  
4.3.5 Assumed Displacement by Generalized Coordinates 105
Contents

4.4 Element Stiffness Relationship for a Beam with Uniform Lateral Load 106
  4.4.1 Finite Element Formulation Based on Virtual Work and Assumed Displacement 106
  4.4.2 Application of Beam Element with Cubic Displacement Function 109
  4.4.3 Final Remarks 110
4.5 Stiffness Relationship for Bar Element – Beam with Axial Force 111
  4.5.1 General 111
  4.5.2 Beam Under Axial Loads 112
  4.5.3 Beam Under Axial and Lateral Loads 113
4.6 Stiffness Relationship for Beam with Bending and Shear Deformation
  4.6.1 Matrix Method 116
  4.6.2 Finite Element Model Based on Timoshenko Beam Theory 117
4.7 Coordinate Transformations 119
  4.7.1 General 119
  4.7.2 Translation 119
4.8 Finite Elements for Linear, Static Structural Analysis 120
4.9 System Stiffness Relationship for Static Problems 122
  4.9.1 General 122
  4.9.2 Global Interpolation Functions 122
  4.9.3 Principle of Virtual Displacements for System 123
  4.9.4 Finite Element Model for System 124
  4.9.5 Example – One-Element Model 125
  4.9.6 Example – Three-Element Model 126
  4.9.7 Buckling Analysis of a Structural System 127
4.10 Dynamic Structural Analysis Models 129
  4.10.1 Dynamic Equilibrium for a Structure with Concentrated Masses and Damping 129
  4.10.2 Dynamic Equilibrium Based on Virtual Work with Consistent Mass and Damping Matrix Formulations 130
  4.10.3 Example – Bar with Free Harmonic Axial Vibrations 132
  4.10.4 Example – Slender Beam with Free Harmonic Bending Vibrations 133
4.11 Modal Analysis 136
  4.11.1 General 136
  4.11.2 Reduction of Modes 137
4.12 Other Approaches for Reducing the Number of DOFs in Elastic Structures 139

5 Stochastic Processes ................................. 142
  5.1 Introduction 142
  5.2 Examples of Stochastic Modeling 142
  5.3 Random Variable, Mean Value, and Variance 146
Contents

5.4 Definition of a Stochastic Process 147
  5.4.1 Example – An Elementary Stochastic Process 148
  5.4.2 Example – A Harmonic Stochastic Process 148
5.5 Joint Probability Distributions 149
5.6 Correlation 150
5.7 Stationary Processes 152
  5.7.1 Example – A Stationary Harmonic Process 154
  5.7.2 Example – A Stationary Process 155
5.8 Ergodic Processes 156
  5.8.1 Example – An Ergodic Harmonic Process 157
  5.8.2 Example – An Ergodic Process 158
5.9 Realizations of Stochastic Processes 159

6 Variance Spectrum ........................................... 161
  6.1 Introduction 161
  6.2 Variance Spectrum 161
  6.3 Units of Variance Spectra 163
    6.3.1 Example – A Realization of a Wave Process 164
  6.4 Examples of Variance Spectra and Autocovariances 167
    6.4.1 Constant Autocovariance 167
    6.4.2 Harmonic Process 169
    6.4.3 Periodic Process 170
    6.4.4 Rectangular Spectrum 170
  6.5 The Variance Spectrum Directly from the Realizations 173

7 Environmental Loads .......................................... 175
  7.1 Introduction 175
  7.2 Hydrodynamic Loads, Added Mass, and Damping 176
    7.2.1 Nonlinear Features of Morison Type Loads 178
    7.2.2 Nonlinear Loads on Large Volume Structures 179
    7.2.3 Effect of Phase Angle on Wave Forces 181
    7.2.4 Mass, Damping, and Stiffness 181
  7.3 Wind Loads 183
  7.4 Ice Loads 186
  7.5 Seismic Loads 189

8 Random Environmental Processes ............................. 191
  8.1 Introduction 191
  8.2 Ocean Waves 191
    8.2.1 Wave Process 191
    8.2.2 Wave Spectra 194
    8.2.3 The Distribution of the Wave Surface Elevation 197
    8.2.4 The Distribution of Wave Crests 198
    8.2.5 The Distribution of Wave Heights 201
### Contents

8.3 Wind ............................. 203
   8.3.1 Wind Speed .................. 203
   8.3.2 Wind Shear and Turbulence 204
   8.3.3 Wind Spectra .......... 206

9 Response Spectrum ................. 209
   9.1 Introduction ................. 209
   9.2 Representation of the Response Process 209
   9.3 Mean Value of the Response Process 210
   9.4 Autocovariance of the Response Process 211
   9.5 Response Spectrum .......... 212
       9.5.1 Example – Response Spectra for a Crane Vessel 213
   9.6 Cross-Covariance .......... 216
   9.7 Cross-Spectrum .......... 217
   9.8 Cross-Spectrum Directly from Realizations 219
   9.9 Spectra and Cross-Spectra of Differentiated Processes 219
   9.10 Coherence Function .......... 222
   9.11 Response to “White Noise” .......... 222
   9.12 Response to a Narrow-Banded Load Process 224
   9.13 Response to Random Wave Loads 227
       9.13.1 Example (Continuation of Example 9.5.1) 227
   9.14 Response Spectra of MDOF Linear Systems 228
       9.14.1 Transfer Function Matrix of an MDOF Response Process 228
       9.14.2 Covariance Spectral Matrix ....... 229

10 Response Statistics ................. 233
   10.1 Introduction ................. 233
   10.2 Average Rate of Level Crossings 233
   10.3 Statistical Distribution of Peaks of a Narrow-Banded Process 237
   10.4 Average Upcrossing Frequency and Statistical
       Distribution of Peaks of a Gaussian Process 238
   10.5 Extreme Values .......... 240
   10.6 Classical Extreme Value Theory ....... 242
   10.7 Extreme Values of Gaussian Processes 244
       10.7.1 Example – (Continuation of Examples 9.5.1 and 9.13.1) 247
       10.7.2 Example – The Crossing Rate of Transformed Processes 248
   10.8 Return Period .......... 249
   10.9 Basic Notions of Fatigue Damage 250

11 Statistics for Nonlinear Problems ....... 252
   11.1 Introduction ................. 252
   11.2 Hydrodynamic Forces on Slender Structures 252
       11.2.1 Morison Equation .......... 252
       11.2.2 Statistics of Morison-Type Wave Forces 253
Contents

14 Equations of Motion ........................................ 320

14.1 Introduction ............................................. 320
14.2 Solution of Equations of Motion ............. 320
  14.2.1 General ............................................. 320
  14.2.2 Eigenvalue Problem ......................... 322
  14.2.3 Frequency Response Method ............... 323
  14.2.4 Formulation and Solution of Frequency-Domain
       Equations ........................................... 325
  14.2.5 Hybrid Frequency- and Time-Domain Models
       ................. 326
  14.2.6 State-Space Formulation ................. 329

15 Numerical Solution Techniques ....................... 331

15.1 Introduction ............................................. 331
15.2 Newmark Methods for SDOF Models .......... 332
  15.2.1 Linear Models ................................. 332
  15.2.2 Nonlinear Models ......................... 334
15.3 Newmark Methods for MDOF Models .......... 336
  15.3.1 Linear Models ................................. 336
  15.3.2 Nonlinear Models ......................... 338
15.4 Runge-Kutta Methods .............................. 339

16 Monte Carlo Methods and Extreme Value Estimation .......... 341

16.1 Introduction ............................................. 341
16.2 Simulation of Stationary Stochastic Processes ......... 341
16.3 Monte Carlo Simulation of Load and Response ....... 342
16.4 Sample Statistics of Simulated Response ............ 342
16.5 Latin Hypercube Sampling ......................... 344
16.6 Estimation of Extreme Response .................. 345
  16.6.1 Peaks-Over-Threshold Method .............. 345
  16.6.2 Gumbel Method ................................ 349
  16.6.3 Naess-Gaidai Method ......................... 350
  16.6.4 The Average Conditional Exceedance Rate Method
       ................................................. 353
  16.6.5 A Comparison of Methods .................. 356
  16.6.6 Combination of Multiple Stochastic Load Effects
       ................................................. 364
  16.6.7 Total Surge Response of a TLP .............. 373

A Integrals .................................................. 385

B Poisson Process ............................................ 386

C Statistical Moments and Cumulants ................. 388

References .................................................. 391

Index ....................................................... 405
Preface

This textbook provides the material for both basic and intermediate modern courses in dynamic analysis of ships and offshore structures. The word “modern” is used to signify that both deterministic and stochastic dynamics are covered. Because the main goal is to provide an introduction to dynamic analysis, the basic elements are described in some detail. A consequence of this is that the majority of the book deals with structures or structural elements that can be modeled or reduced to a single-degree-of-freedom (SDOF) system. However, realizing that multi-degree-of-freedom (MDOF) systems are unavoidable in many practical situations the engineer is likely to meet, and, consequently, that the basic principles for their analysis should be understood, a chapter on linear MDOF systems is included. This is also done to clearly demonstrate the principle of modal decomposition whereby an MDOF system is reduced to a set of uncoupled SDOF systems.

Broadly speaking, a dynamic analysis is carried out in two different ways according to how the loading is specified. If the time-variant loading is given in such a way that we may consider it to be exactly known as a function of time, the same will apply to the response. In such a case, the dynamic analysis is called deterministic. This is in contrast to a stochastic analysis, where the loading is specified using probabilistic concepts. This implies that the corresponding displacements and tensions can only be described in the same way. Even if naturally occurring loading to which a structure is subjected, such as wind and waves, can be claimed to be deterministic, its specification in terms of fundamental physical laws will remain beyond reach for any foreseeable length of time. For such types of loading, a stochastic description has proven to be exceedingly useful.

The first part of the book (Chapters 2–4) describes fundamental aspects of a deterministic dynamic analysis, with emphasis on simple but important dynamic problems relevant for marine structures. The second part (Chapters 5–16) provides a rather extensive introduction to stochastic dynamics of marine structures. Even though the book is focused on marine structures, with a suitable selection of material, it may also serve as a textbook for a more general course in the deterministic and stochastic dynamics of structures.

This book is supported by a Web site (www.cambridge.org/naess) containing numerous problems, many related to ships and offshore structures, that will make it useful not only for students, but also for professional engineers.
Many colleagues and friends offered their comments and suggestions for improving the book manuscript. In particular, the authors want to thank Professors K. M. Mathisen, E. Hjorth-Hansen, G. Moe, L. V. S. Sagrilo, B. J. Leira, S. Haver, N. Saha, and Z. Gao. The help rendered by Huirong Jia, Biao Su, and Mahmoud Etemad is also greatly appreciated.

Figures and illustrations are an important ingredient in a book like this. Most of these were done by Ole Erik Brandrud Naess and Gaute Halvorsen. Their expert assistance is much appreciated.

The authors gratefully acknowledge the support of the Norwegian Research Council through the Centre for Ships and Ocean Structures (CeSOS) at the Norwegian University of Science and Technology. The first author would also like to thank the Norwegian Non-Fiction Writers and Translators Association (NFF) for its support.