

CHAPTER 1

Conceptual Foundations of Capital Market Anomalies

Mozaffar Khan

This book describes unexpected price behavior in equity markets, termed Anomalies, that can potentially be exploited by investors to earn abnormal returns. In capital markets, an anomaly is a deviation from the prediction of the efficient markets theory. The purpose of this chapter is to provide a conceptual framework for understanding the academic research on anomalies and to evaluate whether certain anomalies can be profitably exploited. The chapter begins with a discussion of efficient markets theory, which specifies how assets (specifically stocks) are expected to be priced under a set of ideal or theoretical conditions. The discussion then moves on to anomalies, or price behavior, that is unexpected if markets are efficient. The chapter defines anomalies, discusses explanations for anomalies that have been examined in the academic literature, and concludes by weighing the evidence for these different explanations. Since anomalies yield predictable positive risk-adjusted returns, proper risk measurement is critical to the identification of anomalies. Hence, the appendix to this chapter provides a detailed review of risk measurement and expected return models.¹

¹This chapter was conceived while the author was an assistant professor at the Massachusetts Institute of Technology. He wishes to express his gratitude to the MIT Sloan School of Management and the Carlson School of the University of Minnesota for support. Also Zhaoyang Gu, Prem Jain, Leonid Kogan, Stephannie Larocque, John Douglas Lyon, Hai Lu, Thomas Lys, Steffen Penman, Konstantin Rozanov and Pervin Shroff for valuable comments.

No specific anomaly is discussed in this chapter, because the discussion here is intended to be applicable to all anomalies. It is hoped that, at the end of this chapter, investors will have the conceptual tools necessary to evaluate and understand observed price behavior in general, and the anomalies discussed in the subsequent chapters in particular.

Efficient Markets

The efficient markets theory is usually credited to Fama (1965, 1970), and also has theoretical roots in Samuelson (1965) and Mandelbrot (1966). A market is informationally efficient if prices are, on average, correct, given the publicly available information. Prices react rapidly to new events, and, on average, correctly impound the new information. This characterizes an equilibrium in a competitive market if the following conditions, among others, hold:

- *Structural Knowledge.* Investors are assumed to have complete information about the underlying structure of the return-generating process. For example, investors know the parameters and functional form of the model that governs the stock's returns. Consider what happens when this information is not known for a given stock S . An event may change the risk or expected cash flows of S , but if there is preexisting uncertainty about the parameters of the pricing equation for S , it is difficult to revise the price so that it correctly impounds the new information.
- *Rational Information Processing.* Investors, on average, are assumed to process information in a cognitively unbiased, Bayesian fashion. They are not subject to psychological biases that cause them to over- or underreact to information. Although there may be some investors who are not rational, their trades are unlikely to be correlated, so their irrational trades essentially cancel each other out (noise trading).
- *No Limits to Arbitrage.* Even if the trades of irrational investors are correlated and result in mispricing, rational investors will quickly step in and arbitrage away the mispricing. Absent frictions, arbitrage facilitates market efficiency by quickly eliminating deviations from fundamental values. Frictions that limit arbitrage include transaction costs, short-sale constraints, a limited number of arbitrageurs combined with specialization among arbitrageurs, the absence of close substitutes for the mispriced stock, lingering heterogeneity of investor opinion about the "correct" price for the stock, and bounded investment scalability.

It is useful to keep the preceding assumptions in mind because, when they are violated, they become potential explanations for observed

mispricing. A stock may be mispriced if any combination of these assumptions does not hold. The efficient markets theory is perhaps the single most pervasive organizing principle in finance. Its power lies in:

- The range of phenomena it is capable of explaining and predicting. The average stock at a random point in time is likely fairly priced. If mispricing were rampant and easily identifiable by the average investor, paid investment professionals might be obsolete. Paid investment professionals are more likely needed when mispricing has to be ferreted out of dark corners, than when mispricing exists out in the open.
- The discipline it forces on our thinking. When an ostensibly mispriced stock is identified, it forces us to understand why it is mispriced, or in other words, it forces us to ask why the mispricing signal is expected to be reliable. Investment decisions attempt to anticipate future outcomes, and these outcomes are difficult to predict absent understanding of the reasons for the mispricing.
- The guide it provides to understanding why a stock may be mispriced. This guide is the set of assumptions of the theory outlined previously. The theory then, in essence, tells us which explanations (i.e., assumptions) to explore in attempting to understand why a given stock may be mispriced.

Respect for the efficient markets theory, and an acknowledgement that it sometimes fails (i.e., that mispriced stocks can be identified), can coexist. One need not disdain the theory in the pursuit of anomalies, to which we turn next.

Identifying Anomalies in Capital Markets

Capital market anomalies are deviations from the prediction of efficient markets theory. Such anomalies manifest in predictable nonzero risk-adjusted returns (RAR). A stock with zero risk-adjusted returns provides a fair return for its risk. A stock with positive (negative) risk-adjusted returns provides a more-than-fair (less-than-fair) return for its risk. Investors would like to be long the former and short the latter.

A theory is an approximation of reality. Zero approximation errors are unheard of in practice. According to Kuhn (1962), anomalies are common and expected in every field, and they are an integral part of the routine “puzzle-solving” process of science. Scientists are reluctant to discard a broad theory or paradigm upon discovery of some instances of its falsification (i.e., significant approximation errors). To discard a paradigm, a replacement candidate that better explains at least as wide a range of phenomena is needed. This burden of competition is necessary for robust strains

of theory to emerge. Therefore, subjecting anomalies to healthy skepticism should be seen as part of the normal discovery process of science in which the objective is to develop robust theories (Kuhn 1962), in our case, a robust theory of asset pricing.

There are essentially two steps in identifying anomalies. The first step is identifying a mispricing signal. An example of a mispricing and, hence, an investment signal is the magnitude of a firm's earnings surprise. Firms with extreme positive (negative) quarterly earnings surprise have predictably higher (lower) future returns, so the investment strategy is to go long (short) on stocks of firms with an extreme positive (negative) earnings surprise in order to earn positive returns. This is known as the post-earnings announcement drift (PEAD) anomaly. In subsequent chapters, a number of different mispricing signals (or anomalies) are described.

The second step is evaluating the economic significance and statistical reliability of the mispricing signal. The typical approach is to sort the cross-section of firms into, for example, deciles based on a mispricing signal. For example, firms would be sorted into deciles of earnings surprise in the PEAD strategy, with the top (bottom) decile containing firms with the highest positive (most negative) earnings surprise. The magnitude of the average risk-adjusted return, or alpha, on a portfolio that is long on stocks in one extreme decile, and short stocks in the other extreme decile, is a measure of the economic significance of the mispricing signal. The alpha is the raw return on the portfolio minus the expected return based on the risk of the portfolio.² A long-short portfolio is not necessarily risk-neutral, and, therefore, it is more common to examine alphas, rather than raw returns, to the long-short portfolio. Many anomalies described in the subsequent chapters typically yield alphas of about 10% per year. The costs, such as information, search, and trading costs of the strategy are also typically subtracted from the alpha in practice to arrive at an estimate of the economic significance of an implementable trading strategy based on the mispricing signal.

The statistical reliability of the mispricing signal is measured by how reliably different the trading strategy's alpha is from zero. Consider a strategy that is implemented annually and can be back-tested on 40 years of data. In this case, we would have 40 separate risk-adjusted returns, one for each year the strategy is implemented. We would expect some variation in risk-adjusted returns across the 40 years. If the variation is low relative to the mean risk-adjusted return, the strategy would be considered statistically reliable. In particular, a t-statistic with a p-value less than 5% is the typical criterion for statistical reliability of an alpha.

²Risk adjustment and expected return models are reviewed in detail in the appendix to this chapter.

Explaining Anomalies

The academic literature has pursued several potential explanations for capital markets anomalies.

- One subset of the literature explores whether the anomaly in question is real. The ostensible anomaly may be: an artifact of mismeasured risk; a result of mismeasured statistical reliability; or a result of data snooping.
- Another subset of the literature explores whether anomalies can be explained by rational structural uncertainty, whereby mispricing is a result of uncertainty about the underlying return-generating process (a violation of the first assumption of efficient markets identified in the previous section).
- A third subset of the literature explores whether investors' psychological biases are responsible for mispricing (a violation of the second assumption of efficient markets identified in the previous section).
- A fourth subset of the literature explores whether limits to arbitrage can explain the persistence of mispricing (a violation of the third assumption of efficient markets identified in the previous section).

These explanations are discussed in the following section.

Is the Anomaly Real?

A real anomaly is one that can be profitably exploited by investors to earn statistically reliable and positive risk-adjusted returns. Identifying a real anomaly, therefore, requires ensuring that the risk of the investment strategy is correctly measured (for proper risk adjustment), and that the RARs are statistically reliable and expected to persist out of sample, as discussed in the next section.

RISK MISMEASUREMENT The expected return on a stock is determined in theory by its risk, so if the theory holds, a stock is not expected to have predictably nonzero alphas. Alpha is the difference between the realized return and a model-implied expected return or benchmark. If the benchmark is too low (high), the alpha can appear positive (negative). This is known as the joint hypothesis problem: any test of market efficiency (the proposition that risk-adjusted returns are zero on average) is also jointly a test of the assumed equilibrium model for expected returns. Therefore, a failure of the joint hypothesis could be due to a misspecification of the expected return model, rather than to failure of market efficiency (Fama 1970). For example, a researcher may find a 5% alpha using the CAPM, but the alpha may be

insignificantly different from zero when the Fama and French (1993) model is used. In the appendix to this chapter, risk measurement and expected return models are reviewed in detail.

This literature has a long tradition and continues to be fertile. Researchers develop new expected return models to better explain anomalies. Examples include Fama and French (1993), Lettau and Ludvigson (2001), Campbell and Vuolteenaho (2004), Khan (2008), and Chen, Novy-Marx, and Zhang (2010), among many others. New expected return models have demonstrated success in explaining away some anomalies. Investors are clearly better served by being cognizant of the need for proper risk measurement. If the wrong model is used, a stock considered an attractive buy (i.e., considered to be undervalued) may actually be a poor buy (it may not be undervalued).

STATISTICAL RELIABILITY Some deviations from market efficiency have been debated on the grounds that the abnormal return is not statistically reliable if alternative statistical methods are used. Measuring long-horizon abnormal stock-return performance after corporate events, such as seasoned equity offerings and mergers, among other events, is particularly challenging. This is because of such problems as survival and selection bias, positive skewness in long-horizon returns and cross-correlation of event-firm returns, among others. Papers such as Barber and Lyon (1997), Kothari and Warner (1997), Fama (1998), Lyon, Barber, and Tsai (1999), and Mitchell and Stafford (2000) have suggested that previously reported abnormal stock returns following corporate events may not be abnormal (i.e., that the stock returns are fair compensation for risk once appropriate statistical issues are addressed). Essentially, these papers point out a statistical Type I error: The null hypothesis of zero abnormal returns is falsely rejected. In response, some authors point out a potential statistical Type II problem: The null hypothesis of zero abnormal returns after corporate events is false, but existing models and tests suffer from low statistical power to detect abnormal stock return performance (Loughran and Ritter 2000; Nekrasov, Singh, and Shroff 2010).

DATA-SNOOPING Lo and MacKinlay (1990) reiterate the inferential hazard that results from empirically overexploring, or mining, a given dataset such as the return history of all traded stocks. This data-snooping problem is also statistical in nature, but it relates to the manner in which the community of scientists collectively discovers knowledge. In contrast, the statistical issues described in the previous paragraph relate more to one particular researcher's choice of test methodology. In a sense, data snooping is a metastatistical problem.

The problem essentially is that researchers using a dataset might find an accidental pattern in the data; that is, they find an accidental pattern, as opposed to accidentally finding a real pattern. A real pattern is one for which there exists an economic rationale or theory, and a real pattern is expected to persist out of sample. An accidental pattern should be ignored, but given the difficulty in distinguishing an accidental from a real pattern, subsequent researchers may be attracted to the unusual finding or anomaly and will select the sample to be studied based on the previous empirical finding (Lo and MacKinlay 1990). This results in a selection bias that can lead to spurious inferences (see also Leamer 1978). A number of specific anomalies have turned out in subsequent research to be more apparent than real, consistent with data-snooping biases (Schwert 2003).

In summary, a large stream of the literature has studied whether numerous specific anomalies are, in fact, anomalies or whether they are consistent with efficient markets. Several anomalies have been explained away once appropriate risk and statistical corrections are made. However, many anomalies have yet to be explained by these methods. Useful surveys of efficient market explanations for anomalies include Fama (1998), Schwert (2003), and Ross (2005).

Rational Structural Uncertainty

Efficient markets theory assumes investors have complete knowledge of the underlying statistical processes that generate returns; that is, it assumes they know the parameters of the pricing equation for each security. All investors also have homogenous opinions about these parameters. In practice, this assumption is unlikely to hold for young firms with a short history and few assets in place. Empirically, such firms are exactly the ones whose stock returns are generally considered anomalous (e.g., Fama and French 1993). Therefore, such firms may be mispriced if investors have incomplete information about valuation parameters (e.g., Merton 1987) or uncertainty about these parameters (Brav and Heaton 2002). Mispricing generated by such rational structural uncertainty can be hard to distinguish empirically from mispricing generated by behavioral or cognitive biases, since rational structural uncertainty and behavioral theories generate similar testable predictions (Brav and Heaton 2002).

It is important to note that anomalous stock returns are not necessarily due to cognitively biased investors. In this section, we take as given that a specific anomaly is real rather than apparent; that is, that the anomaly is not subject to the problems previously identified. The question is what generates the real anomaly. Rational structural uncertainty theories highlight that it is possible for investors to process information rationally, and yet to

misprice stocks if they have incomplete information or uncertainty about valuation parameters.

Behavioral Finance and Limits to Arbitrage

Efficient markets theory assumes that investors process information rationally, without any cognitive biases. Behavioral finance refers to the class of theories that relax the rationality assumption and propose that the behavior of security prices is better explained by investors' behavioral or cognitive biases. Biases such as sentiment, overconfidence, biased self-attribution, conservatism, and a representativeness heuristic generate underreaction and overreaction to information, which manifests in underpricing and overpricing (DeBondt and Thaler 1985; Lee, Shleifer, and Thaler 1991; Lakonishok, Shleifer, and Vishny 1994; Barberis, Shleifer, and Vishny 1998; Daniel, Hirshleifer, and Subrahmanyam 1998; Hong and Stein 1999). Other biases, such as loss aversion (Kahneman and Tversky 1979) generate a reluctance to sell losing stocks, as documented in Odean (1998) for example.

Efficient markets theory also assumes that, even if some investors are irrational and generate mispricing, rational investors will quickly arbitrage the mispricing away. Behavioral finance, in response, highlights numerous limits to arbitrage, such as the following, which allow a wedge between fundamental and observed values to persist.

- *Transaction Costs.* Arbitrageurs evaluate profits after trading costs, so trading costs of $X\%$ can sustain mispricing of the same magnitude. In practice, trading costs for large investors are too small to explain the substantial magnitudes (10% to 30% per year) reported for some anomalies.
- *Short Sale Constraints.* These include the cost of borrowing and locating the stock from securities lenders. The direct borrowing costs are negligible (e.g., D'Avolio 2002), but locating the stock can be a substantial barrier. Another impediment is the risk that a borrowed stock may be recalled by the lender when the stock price has gone up, before the borrower has had a chance to earn a profit. Small, young, and illiquid stocks are difficult to locate, and these are the stocks for which mispricing is typically observed empirically. Short sale constraints have the potential to explain sustained overpricing but not underpricing of certain stocks.

These constraints apply to investors who are permitted to short sell. Many large institutional investors, such as mutual funds, are barred by charter from short selling. Although this may limit the number and type of investors who can bring adverse information to the market through

short selling, it is unlikely a significant constraint, given that there are many other large investors, such as hedge funds, which do not face short selling restrictions.

- *Arbitrageur Presence.* Professional arbitrageurs specialize in certain stocks that they follow closely based on their expertise and profit opportunities. Combined with a limited numbers of arbitrageurs, this implies there are many stocks with limited or no arbitrageur presence.
- *Absence of Close Substitutes.* Arbitrageurs hedge their position in a mispriced stock by simultaneously taking an offsetting position in a close substitute. Many stocks or portfolios of securities do not have close substitutes, which makes arbitrage risky.
- *Lingering Differences in Investor Opinion.* Even if a stock has a perfect substitute, the arbitrageur faces the risk that the mispricing does not correct within his investment horizon. If differences in investor opinion about the fundamental value of the stock linger or worsen, the arbitrageur may be unable to profitably close his position within his investment horizon. As John Maynard Keynes pithily observed, the market can stay irrational longer than one can stay solvent (as cited in Lowenstein 2001). This risk further limits arbitrage.
- *Unscalable Opportunity.* The mispriced stock may not be available in sufficient numbers to allow the arbitrageur to recover fixed costs. This is related to arbitrageur presence previously described.

Combining cognitive biases with limits to arbitrage, behavioral finance theories have sought to explain numerous efficient markets anomalies.

Irrational behavior also motivates reaction to noninformation, as inferred from both the first and second moment of returns (average and volatility). For example, Shiller (1981) and Roll (1988) suggest returns are too volatile to be explained by economy-wide, industry, or firm-specific fundamental news. Shleifer (1986), Greenwood (2005), Coval and Stafford (2007), and Khan, Kogan, and Serafeim (2011) suggest average returns to individual stocks change in response to uninformed demand shocks. In the latter literature, the price movements are not driven by individual investors' behavioral biases but, rather, by institutional constraints that lead to large uninformed stock purchases or sales. Therefore, this evidence is not so much for behavioral biases as it is against efficient markets, but it does rely on limited arbitrage for the sustained mispricing.

The volume of the behavioral finance literature and the academic reputation of many of its proponents suggest it is the most popular challenger to efficient markets as an explanation for stock price behavior. Useful surveys of behavioral explanations for anomalies can be found in Thaler (1993) and Shleifer (2000).

Anomalies: Weighing the Evidence

The weight of the evidence in the literature both for and against efficient markets is impressive. This makes it difficult to draw unqualified conclusions about which theory best describes stock price behavior. The difficulty stems from the fact that, for many anomalies, the evidence is consistent with both rational and behavioral explanations. It is perhaps safe to say that, currently, no one theory completely describes all price behavior.³ This need not be unduly distressing, for a few reasons. First, other fields face similar conflicts. In physics, for example, there is one theory for the very large (relativity) and a different theory for the very small (quantum mechanics). In a sense there are two theories of stock price behavior, but the challenge is to discriminate between, and predict, instances when each theory is expected to hold. Ultimately, of course, the Holy Grail, as in physics, is to develop one unified theory. Second, there is a sense of order imparted by markets that are, on average, efficient, yet a sense of hope for investors imparted by occasional deviations from efficiency. Deviations from efficiency stimulate private information search (Grossman and Stiglitz 1980) and competition among investment professionals, activities that make the market more efficient.

Fortunately there is a less controversial answer to the question of whether anomalies are real or apparent. Real anomalies exist, in the sense that they are resistant to efficient market explanations and present opportunities for abnormal profits. However, not all anomalies are real: In some cases the profits are not abnormal but are simply appropriate compensation for risk. The purpose of this book is to describe the state of the academic literature in selected anomalies. The purpose of this chapter is to provide investors and investment professionals the conceptual tools needed to discriminate between real and apparent anomalies. Hopefully, the reader is thus armed as the journey begins in the next chapter.

Appendix 1.1: Risk and Expected-Return Models

In this appendix, models of expected returns are reviewed, in which the expected return is based on the risk of the asset (or the risk in its future cash flows). Proper risk adjustment is critical in identifying anomalies because, for example, a portfolio that really has zero RARs may appear to have positive

³Another theory of price behavior besides those discussed here is proposed in Lo (2004, 2005), and is known as the Adaptive Markets Hypothesis.

RARs if an inappropriate expected return model is used. In general, risk and expected return models are important for the following reasons.

- Portfolio Selection – Value: Because the expected return, r , is an input in all valuation models, investors need an accurate assessment of risk and expected return to determine whether a stock is fairly priced or fairly valued. Investors will buy (sell) stocks that are undervalued (overvalued) by the rest of the market.
- Portfolio Selection – Risk: Investment managers need accurate portfolio risk assessments, to ensure that it meets the risk tolerance of their clients.
- Performance Evaluation: Investors need accurate portfolio risk assessments, to evaluate the performance of their investment managers. Given the risk-return trade-off, investors evaluate return performance relative to the assumed risk, or, in other words, investors care about risk-adjusted returns rather than raw returns. Consider two investment managers who both earn a 10% raw return on their respective portfolios. However, if one portfolio is riskier than the other, investors will require higher returns on the riskier portfolio. Using risk-adjusted returns to evaluate their performance would yield the correct conclusion that the manager of the less risky portfolio outperformed the manager of the riskier portfolio. Investors are expected to reward managers who produce positive risk-adjusted returns, and punish those who produce negative risk-adjusted returns.

The expected return on any asset is the sum of the risk-free rate and the asset's risk premium. The risk premium on any asset can be thought of as the price of risk multiplied by the quantity of risk. The price of risk is more precisely the required return per unit of risk, while the quantity of risk is the asset's number of units of risk. For example, the unit of risk (or the quantity of risk) could be the asset's CAPM beta,⁴ and the price of risk would be the required return on a unit beta asset. The risk premium on a riskier asset, which has a beta of 2 for example, would be 2 times the price of risk. The price of risk is the same for all assets, whereas the quantity of risk varies across assets. This suggests that, to calculate expected returns, we need to start by thinking about how to measure the risk of a stock.

Investors prefer a smooth consumption stream. Stocks that smooth out their consumption stream (e.g., stocks that are negatively correlated with consumption) are less risky than stocks that amplify the volatility of their consumption stream (e.g., stocks that are highly positively correlated with

⁴A stock's CAPM beta is the coefficient from a regression of the stock's excess returns on the market excess return. Excess return is the return in excess of the risk-free rate.

consumption). Hence, the fundamental measure of risk is covariance with consumption. This is formally derived by setting up a utility maximization program. Solving for the first-order conditions yields a general expression for the expected return on any asset (Cochrane 2001):

$$E(r - R_f) = -R_f \text{Cov}(SDF, r) \quad (1.1)$$

In equation (1.1), SDF is the stochastic discount factor or pricing kernel, and it is approximately equal to consumption growth. Equation (1.1) says that the expected risk premium on any asset is a function of its return covariance with the SDF. A riskless asset is one whose return is known ex ante with certainty, and since its covariance with the SDF is zero, the expected return on the riskless asset is the risk-free rate R_f . A risky asset is negatively correlated with the SDF, and, hence, has a positive risk premium. A hedge is positively correlated with the SDF, and, hence, has a negative risk premium (i.e., an expected return lower than the risk-free rate).

Equation (1.1) is not empirically estimable as is, because it tells us neither the empirical proxies to use for the SDF nor the functional form of the relation between the SDF and these proxies. This task is left to economic models such as the capital asset pricing model (CAPM), the consumption-based capital asset pricing model (CCAPM), the intertemporal capital asset pricing model (ICAPM), and others. These models tell us where the SDF comes from (i.e., what risk factors proxy for the SDF), and also propose a linear relation between the SDF and these risk factors. Hence, expected return models such as the CAPM, ICAPM, APT, Fama-French, and others differ basically in terms of the number and identity of risk factors they propose.

Given a set of risk factors, and a linear relation between the SDF and these risk factors, equation (1.1) is rewritten in most academic work as:

$$E(r - R_f) = \sum (\beta_j \lambda_j) \quad (1.2)$$

In equation (1.2), j is the number of risk factors, β_j is the beta of a given stock with respect to the j th risk factor, and λ_j is the price of the j th risk factor (i.e., the expected risk premium on a stock with $\beta_j = 1$). Equation (1.2) is one representation of equation (1.1). The Mean-Variance frontier is another representation of the same.⁵ In other words, equations (1.1) and (1.2) and the Mean-Variance frontier are equivalent. This is useful to know because it emphasizes the common root of all asset pricing models: These are not different models, but simply different representations of a

⁵Portfolios on the upper part of the Mean-Variance frontier are negatively correlated with the SDF, whereas portfolios on the lower frontier are positively correlated with the SDF. An investor may wish to place a portion of her savings in portfolios on the lower frontier, for hedging purposes.

common underlying model (Cochrane 2001). The links between them were first developed in Roll (1977), Ross (1978), and Hansen and Richard (1987). Cochrane (2001) presents an accessible and comprehensive development.

Capital Asset Pricing Model

The CAPM of Sharpe (1964) and Lintner (1965) is the most widely known asset pricing model. It can be derived as a 2-period model (i.e., assuming investors liquidate the investment in the second period) under certain assumptions, but also as an infinite period model (i.e., the stock is held to infinity) assuming that the investment opportunity set (IOS) is nonstochastic. A nonstochastic IOS means that, for example, expected returns and return volatilities are not time varying. The basic result of the CAPM is that the SDF is a linear function of the investors' total wealth. In practical applications, the proxy for total wealth is assumed to be the return on the market portfolio of equities, R_m . This then gives us the familiar expression for the CAPM:

$$E(r) = R_f + \beta(R_m - R_f) \quad (1.3)$$

Equation (1.3) is one version of equation (1.2), with $j = 1$ and $\lambda_j = E(R_m - R_f)$. Roll (1977) suggests tests of the CAPM may be sensitive to the proxy used for investors' total wealth, because a broad equity portfolio does not represent a claim on all tradable wealth (see also Mayers 1973). Stambaugh (1982) uses a portfolio of equities, corporate and Treasury bonds, residential real estate, and other assets as a proxy for total wealth, and shows that empirical tests of the CAPM are insensitive to the composition of the market portfolio. Therefore, the use of a broad portfolio of equities only, as a proxy for total wealth in equation (1.3), has survived in common practice. The CAPM continues to be widely used and taught, despite much empirical evidence that its ability to explain the cross-section of stock returns is very poor (e.g., Fama and French 1992). Its continued use could be due to its theoretical intuition and ease of empirical implementation.

Equation (1.3) shows the static, or unconditional, CAPM in which the beta and risk premium do not vary over time. Conditional CAPM specifications allow variation in the beta and risk premium over the business cycle. This is because the price of risk, or required return per unit beta, is expected to increase in uncertain economic times. Risk, or beta, is also expected to increase in economic downturns because, for example, financial and operating leverage cannot be adjusted instantaneously. Empirical evidence on the performance of the conditional CAPM, relative to the unconditional CAPM, in explaining the cross-section of expected returns is mixed (e.g., Jagannathan and Wang 1996; Lettau and Ludvigson 2001; Lewellen and Nagel 2006; Roussanov 2010).

Intertemporal Capital Asset Pricing Model

The ICAPM of Merton (1973) models long-lived investors with stochastic variation in investment opportunities. The model suggests that investors care about not only current wealth, as in the CAPM, but also about future investment opportunities. Future investment opportunities are poor if expected returns decline, signaling that investors' capital will be less productive in the future. This suggests more will need to be saved to grow to a given target amount in the future, thereby reducing consumption today. However, a decline in expected returns is not unambiguously bad news: Because the discount rate declines, it raises the current value of the investor's portfolio. The net effect of a decline in expected returns could be bad news for a long-horizon investor, who cares about long-horizon returns, but good news for a short-horizon investor who intends to consume most of his capital in the near future.

In the ICAPM, future investment opportunities are riskier if future return volatilities are expected to increase. The ICAPM predicts that investors will try to hedge against adverse shocks to current wealth as in the CAPM, and also against adverse shocks to the mean and variance of future investment opportunities. In empirical implementation, the ICAPM can also be expressed as a linear function of state variables or risk factors. These risk factors predict changes in investment opportunities.

Fama and French (1993) developed a popular empirical model in which expected returns are a linear function of returns on the market portfolio and size and book-to-market risk factors. This model is now a workhorse in empirical academic research because of its power in explaining the cross-section of expected stock returns. Empirical evidence suggests the size and book-to-market factors predict changes in future investment opportunities (Liew and Vassalou 2000; Vassalou 2003; Li, Vassalou, and Xing 2006; Petkova 2006), indicating the Fama and French (1993) model is an ICAPM-type model. Campbell and Vuolteenaho (2004) and Khan (2008) also test ICAPM models using macroeconomic risk factors and valuation spreads—term structure and value spreads—that predict changes in future investment opportunities. These authors present evidence that their models explain a substantial portion of the cross-sectional variation in expected returns.

Arbitrage Pricing Theory

The arbitrage pricing theory (APT) of Ross (1976) observes that there is substantial common movement in stocks' returns. The sources of this comovement are called factors. Stocks co-move because they are exposed to or correlated with these factors. The portion of stock returns uncorrelated

with these factors is called the idiosyncratic return. If returns are a linear function of these factors, the idiosyncratic returns are on average zero and uncorrelated with the factors, and the law of one price holds, then the SDF in equation (1.1) can be written as a linear function of these factors (Cochrane 2011). In other words, the factors price all assets because any stock return can be synthesized by a portfolio of the factors. The idiosyncratic return is not expected to be priced (i.e., to be compensated by a risk premium) since it is diversifiable by investors holding portfolios of stocks.

The APT does not tell us the identity or number of factors. For this, we turn to statistical techniques, such as factor analysis, or to economic theory. The latter suggests macroeconomic variables related to the business cycle as risk factors. One empirical example of a linear factor specification based on the APT is Chen, Roll, and Ross (1986), who use the term *spread*, the default spread, unexpected inflation, and industrial production as risk factors.

Production-Based Models

The CAPM and ICAPM were developed by considering the optimizing behavior of consumers or investors. Production-based models, in contrast, solve for the first-order conditions of firms optimizing their production-investment decision. A firm can invest in physical assets to produce more goods next period, or it can invest in financial assets. At the margin, the two rates of return must be equal. Hence we can solve for the expected return on financial assets (e.g., stocks) by solving the firm's production-investment optimization. Production-based models are not distinct from consumption-based models. They are simply the other side of the same coin, but they lead to useful insights and testable restrictions on expected returns (Cochrane 1991). Another advantage is that investments are more variable and cyclical than consumption, and since stock returns are highly variable and cyclical, variation in investments rather than variation in consumption is likely to have higher explanatory power for stock return variation.

Production-based asset pricing was first developed, along with supporting evidence, by Cochrane (1991). Lamont (2000) provides additional evidence, as does Kogan (2004), who also extends the theory. A number of cross-sectional expected return models have been motivated by the production-based theory. Cochrane (1996), an early example, uses residential and nonresidential gross fixed investment returns as factors, and reports that this model performs about as well as the CAPM and the Chen, Roll, and Ross (1986) model in explaining cross-sectional variation in stock returns. Subsequent models perform better. Li, Vassalou, and Xing (2006) specify investment growth rates in four sectors as risk factors: households, nonfarm nonfinancial corporate business, nonfarm noncorporate business, and financial business. Their model performs as well as the Fama and

French (1993) and Lettau and Ludvigson (2001) models. Chen, Novy-Marx, and Zhang (2010) specify three risk factors—excess returns on the market portfolio, an investment factor-mimicking portfolio, and a profitability factor-mimicking portfolio—and they report that their model outperforms the Fama and French (1993) model in explaining the cross-section of stock returns. Their model also explains a number of empirical regularities previously considered anomalous. In summary, production-based asset pricing has resulted in some promising cross-sectional models of expected returns.

Firm-Specific Expected Return Estimates

The models described in the preceding section are typically used to estimate expected returns for portfolios of stocks rather than for individual stocks. This is because expected return estimates for individual stocks are very noisy or imprecise (Fama and French 1997), but estimation noise is lower for portfolios of stocks. There are two reasons for imprecise firm-specific estimates of expected returns: (1) reliable estimation requires a longer time series of data than is available for many firms, and (2) individual stock betas are likely more variable over time, which introduces further uncertainty in estimation.

In many investment applications, a firm-specific expected return estimate is not required. For example, a portfolio manager evaluating an investment signal, say the book-to-price ratio (B/P), is interested in the expected return on high and low B/P portfolios. Subtracting the model-implied expected return from the realized average return yields an estimate of the strategy's alpha. Similarly, for performance evaluation, the expected return (and subsequently alpha) on the portfolio of stocks under management is needed. Where a firm-specific expected return estimate for some stock *S* is required, one can estimate the expected return for a portfolio of stocks that are matched to *S* on various characteristics such as size, book-to-market, industry, and other variables, and then use the expected return on this portfolio as the expected return for *S*.

Implied Cost of Capital

The price of a stock is a function of its expected cash flows and its discount rate. If we take the observed market price of a stock as the true or accurate price, and we have estimates of expected cash flows, we can calculate the discount rate that forces the pricing equation to hold. This inferred discount rate is called the implied cost of capital (ICOC). Examples of this approach to calculating expected returns include Gebhardt, Lee, and Swaminathan (2001) and Ohlson and Juettner-Nauroth (2005). However, the validity of ICOC estimates as measures of expected return is unclear, because empirical

evidence on the ability of ICOC estimates to predict future stock returns is mixed (Easton and Monahan 2005; Botosan, Plumlee, and Wen 2011).

In summary, given that there is a trade-off between risk and expected return, accurately measuring risk is important in order to accurately price stocks, calibrate portfolio risk, and measure investment performance. Therefore, a large part of the academic literature is devoted to increasing our understanding of risk and expected return, as surveyed in this appendix.

References

- Barber, Brad, and John Lyon. 1997. Detecting long-run abnormal stock returns: The empirical power and specification of test statistics. *Journal of Financial Economics* 43: 341–372.
- Barberis, Nicholas, Andrei Shleifer, and Robert Vishny. 1998. A model of investor sentiment. *Journal of Financial Economics* 49: 307–343.
- Botosan, Christine, Marlene Plumlee, and He Wen. 2011. The relation between expected returns, realized returns and firm risk characteristics. Forthcoming, Contemporary Accounting Research.
- Brav, Alon and J. B. Heaton. 2002. Competing theories of financial anomalies. *Review of Financial Studies* 15: 575–606.
- Campbell, John, and Tuomo Vuolteenaho. 2004. Bad beta, good beta. *American Economic Review*, 94: 1249–275.
- Chen, Long, Robert Novy-Marx, and Lu Zhang. 2010. An alternative three-factor model. Working paper, Washington University, University of Chicago, and Ohio State.
- Chen, Nai-fu, Richard Roll, and Stephen Ross. 1986. Economic forces and the stock market. *Journal of Business* 59: 383–403.
- Cochrane, John. 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance* 46: 207–234.
- Cochrane, John. 1996. A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy* 104: 572–621.
- Cochrane, John. 2001. *Asset Pricing*. Princeton, NJ: Princeton University Press.
- Coval, Joshua, and Erik Stafford. 2007. Asset firesales (and purchases) in equity markets. *Journal of Financial Economics* 86: 479–512.
- Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. Investor psychology and security market under- and overreactions. *Journal of Finance* 53: 1839–1885.
- D’Avolio, Gene. 2002. The market for borrowing stocks. *Journal of Financial Economics* 66: 271–306.

- DeBondt, Werner, and Richard Thaler. 1985. Does the stock market overreact? *Journal of Finance* 40: 793–805.
- Easton, Peter, and Steven Monahan. 2005. An evaluation of the reliability of accounting-based measures of expected returns: A measurement error perspective. *The Accounting Review* 80: 501–538.
- Fama, Eugene. 1965. The behavior of stock market prices. *Journal of Business* 38: 34–105.
- Fama, Eugene. 1970. Efficient capital markets: A review of theory and empirical work. *Journal of Finance* 25: 383–417.
- Fama, Eugene. 1998. Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics* 49: 283–306.
- Fama, Eugene, and Kenneth French. 1992. The cross-section of expected stock returns. *Journal of Finance* 47: 427–465.
- Fama, Eugene, and Kenneth French. 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33: 3–56.
- Fama, Eugene, and Kenneth French. 1997. Industry costs of equity. *Journal of Financial Economics* 43: 153–193.
- Gebhardt, William, Charles Lee, and Bhaskaran Swaminathan. Toward an implied cost of capital. *Journal of Accounting Research* 39: 135–176.
- Greenwood, Robin. 2005. Short- and long-term demand curves for stocks: Theory and evidence on the dynamics of arbitrage. *Journal of Financial Economics* 75: 607–649.
- Grossman, Sanford, and Joseph Stiglitz. 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70: 393–408.
- Hansen, Lars Peter, and Scott Richards. 1987. The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica* 55: 587–614.
- Hong, Harrison, and Jeremy Stein. 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance* 54: 2143–2184.
- Jagannathan, Ravi, and Zhenyu Wang. 1996. The conditional CAPM and the cross-section of expected returns. *Journal of Finance* 51: 3–53.
- Kahneman, Daniel, and Amos Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* 47: 263–291.
- Khan, Mozaffar. 2008. Are accruals mispriced? Evidence from tests of an intertemporal capital asset pricing model. *Journal of Accounting and Economics*, 45: 55–77.
- Khan, Mozaffar, Leonid Kogan, and George Serafeim. 2011. Mutual fund trading pressure: Firm-level stock price impact and the timing of SEOs. Forthcoming, *Journal of Finance*.
- Kogan, Leonid. 2004. Asset prices and real investment. *Journal of Financial Economics* 73: 411–431.

- Kothari, S. P., and Jerold Warner. 1997. Measuring long horizon security price performance. *Journal of Financial Economics* 43: 301–339.
- Kuhn, Thomas. 1962. *The structure of scientific revolutions*. Chicago: University of Chicago Press.
- Lakonishok, Josef, Andrei Shleifer, and Robert Vishny. 1994. Contrarian investment, extrapolation, and risk. *Journal of Finance* 49: 1541–1578.
- Lamont, Owen. 2000. Investment plans and stock returns. *Journal of Finance* 55: 2719–2745.
- Leamer, Edward. 1978. *Specification searches*. New York: Wiley.
- Lee, Charles, Andrei Shleifer, and Richard Thaler. 1991. Investor sentiment and the closed-end fund puzzle. *Journal of Finance* 46: 75–110.
- Lettau, Martin, and Sydney Ludvigson. 2001. Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109: 1238–1287.
- Lewellen, Jonathan, and Stefan Nagel. 2006. The conditional CAPM does not explain asset pricing anomalies. *Journal of Financial Economics* 82: 289–314.
- Li, Qing, Maria Vassalou, and Yuhang Xing. 2006. Sector investment growth rates and the cross-section of equity returns. *Journal of Business* 79: 1637–1665.
- Liew, Jimmy, and Maria Vassalou. 2000. Can book-to-market, size, and momentum be risk factors that predict economic growth? *Journal of Financial Economics* 57: 221–245.
- Lintner, John. 1965. The valuation of risky assets and the selection of risky investment in stock portfolios and capital budgets. *Review of Economics and Statistics* 47: 13–37.
- Lo, Andrew. 2004. The adaptive markets hypothesis: market efficiency from an evolutionary perspective. *Journal of Portfolio Management* 30: 15–29.
- Lo, Andrew. 2005. Reconciling efficient markets with behavioral finance: The adaptive markets hypothesis. Working paper, MIT.
- Lo, Andrew, and A. Craig MacKinlay. 1990. Data-snooping biases in tests of financial asset pricing models. *Review of Financial Studies* 3: 431–467.
- Loughran, Tim, and Jay Ritter. 2000. Uniformly least powerful tests of market efficiency. *Journal of Financial Economics* 55: 361–389.
- Lowenstein, Roger. 2001. *When genius failed*. London: Fourth Estate.
- Lyon, John, Brad Barber, and Chi-Ling Tsai. 1999. Improved methods for tests of long-run abnormal stock returns. *Journal of Finance* 59: 165–201.
- Mandelbrot, Benoit. 1966. Forecasts of future prices, unbiased markets and martingale models. *Journal of Business* 39: 242–255.

- Mayers, David. 1973. Non-marketable assets and the determination of capital asset prices in the absence of a riskless asset. *Journal of Business* 46: 258–267.
- Merton, Robert C. 1973. An intertemporal capital asset pricing model. *Econometrica* 41: 867–887.
- Merton, Robert C. 1987. A simple model of capital market equilibrium with incomplete information. *Journal of Finance* 42: 483–510.
- Mitchell, Mark, and Erik Stafford. 2000. Managerial decisions and long-term stock price performance. *Journal of Business* 73: 287–329.
- Nekrasov, Alexander, Pervin Shroff, and Rajdeep Singh. 2010. Tests of long-term abnormal performance: analysis of power. Working paper, University of Minnesota and UC-Irvine.
- Odean, Terence. 1998. Are investors reluctant to realize their losses? *Journal of Finance* 53: 1775–1798.
- Ohlson, James, and Beate Juettner-Nauroth. 2005. Expected EPS and EPS growth as determinants of value. *Review of Accounting Studies* 10: 349–365.
- Petkova, Ralitsa. 2006. Do the Fama-French factors proxy for innovations in predictive variables? *Journal of Finance* 61: 581–612.
- Roll, Richard. 1977. A critique of the asset pricing theory's tests: Part I. *Journal of Financial Economics* 4: 129–176.
- Roll, Richard. 1988. R^2 . *Journal of Finance* 43: 541–566.
- Ross, Stephen. 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13: 341–360.
- Ross, Stephen. 1978. A simple approach to the valuation of risky streams. *Journal of Business* 51: 453–475.
- Ross, Stephen. 2005. *Neoclassical finance*. Princeton, NJ: Princeton University Press.
- Roussanov, Nikolai. 2010. Composition of wealth, conditioning information and the cross-section of stock returns. Working paper, University of Pennsylvania.
- Samuelson, Paul. 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6: 41–49.
- Schwert, G. William. 2003. Anomalies and market efficiency. In *Handbook of the Economics of Finance*, ed. Constantinides, Harris, and Stulz. New York: Elsevier.
- Sharpe, William. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19: 425–442.
- Shiller, Robert. 1981. Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71: 421–436.
- Shleifer, Andrei. 1986. Do demand curves for stocks slope down? *Journal of Finance* 41: 579–590.

- Shleifer, Andrei. 2000. *Inefficient markets: An introduction to behavioral finance*. New York: Oxford University Press.
- Stambaugh, Robert. 1982. On the exclusion of assets from tests of the two-parameter model. *Journal of Financial Economics* 10: 237–268.
- Thaler, Richard (ed.). 1993. *Advances in behavioral finance*. New York: Russell Sage.
- Vassalou, Maria. 2003. News related to future GDP growth as a risk factor in equity returns. *Journal of Financial Economics* 68: 47–73.

Go to <http://hema.zacks.com> for abstracts and links to papers.

