

## Modern Particle Physics

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Unique in its coverage of all aspects of modern particle physics, this textbook provides a clear connection between the theory and recent experimental results, including the discovery of the Higgs boson at CERN. It provides a comprehensive and self-contained description of the Standard Model of particle physics suitable for upper-level undergraduate students and graduate students studying experimental particle physics.

Physical theory is introduced in a straightforward manner with step-by-step mathematical derivations throughout. Fully worked examples enable students to link the mathematical theory to results from modern particle physics experiments. End-of-chapter exercises, graded by difficulty, provide students with a deeper understanding of the subject.

Online resources available at [www.cambridge.org/MPP](http://www.cambridge.org/MPP) feature password-protected fully worked solutions to problems for instructors, numerical solutions and hints to the problems for students and PowerPoint slides and JPEGs of figures from the book.

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MARK THOMSON

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**To**  
**Sophie, Robert and Isabelle**  
*for their love, support and endless patience*

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## Preface

The Standard Model of particle physics represents one of the triumphs of modern physics. With the discovery of the Higgs boson at the LHC, all of the particles in the Standard Model have now been observed. The main aim of this book is to provide a broad overview of our current understanding of particle physics. It is intended to be suitable for final-year undergraduate physics students and also can serve as an introductory graduate-level text. The emphasis is very much on the modern view of particle physics with the aim of providing a solid grounding in a wide range of topics.

Our current understanding of the sub-atomic Universe is based on a number of profound theoretical ideas that are embodied in the Standard Model of particle physics. However, the development of the Standard Model would not have been possible without a close interplay between theory and experiment, and the structure of this book tries to reflect this. In most chapters, theoretical concepts are developed and then are related to the current experimental results. Because particle physics is mostly concerned with fundamental objects, it is (in some sense) a relatively straightforward subject. Consequently, even at the undergraduate level, it is quite possible to perform calculations that can be related directly to the recent experiments at the forefront of the subject.

### Pedagogical approach

In writing this textbook I have tried to develop the subject matter in a clear and accessible manner and thought long and hard about what material to include. Whilst the historical development of particle physics is an interesting topic in its own right, it does not necessarily provide the best pedagogical introduction to the subject. For this reason, the focus of this book is on the contemporary view of particle physics and earlier experimental results are discussed only to develop specific points. Similarly, no attempt is made to provide a comprehensive review of the many experiments, instead a selection of key measurements is used to illustrate the theoretical concepts; the choice of which experimental measurements to include is primarily motivated by the pedagogical aims of this book.

This textbook is intended to be self-contained, and only a basic knowledge of quantum mechanics and special relativity is assumed. As far as possible, I have tried to derive everything from first principles. Since this is an introductory textbook, the

mathematical material is kept as simple as possible, and the derivations show all the main steps. I believe that this approach enables students relatively new to the subject to develop a clear understanding of the underlying physical principles; the more sophisticated mathematical trickery can come later. Calculations are mostly performed using helicity amplitudes based on the explicit Dirac–Pauli representation of the particle spinors. I believe this treatment provides a better connection to the underlying physics, compared to the more abstract trace formalism (which is also described). Some of the more-challenging material is included in optional *starred* sections. When reading these sections, the main aim should be to understand the central concepts, rather than the details.

The general structure of this book is as follows: Chapters 1–5 introduce the underlying concepts of relativistic quantum mechanics and interaction by particle exchange; Chapters 6–12 describe the electromagnetic, strong and weak interactions; and Chapters 13–18 cover major topics in modern particle physics. This textbook includes an extensive set of problems. Each problem is graded according to the *relative* time it is likely to take. This does not always reflect the difficulty of the problem and is meant to provide a guide to students, where for example a shorter graded problem should require relatively little algebra. Hints and outline solutions to many of the problems are available at [www.cambridge.org/MPP](http://www.cambridge.org/MPP).

### For instructors

This book covers a wide range of topics and can form the basis of a long course in particle physics. For a shorter course, it may not be possible to fit all of the material into a single semester and certain sections can be omitted. In this case, I would recommend that students read the introductory material in Chapters 1–3 as preparation for a lecture course. Chapters 4–8, covering the calculations of the  $e^+e^- \rightarrow \mu^+\mu^-$  annihilation and  $e^-p$  scattering cross sections, should be considered essential. Some of the material in Chapter 9 on the quark model can be omitted, although not the discussion of symmetries. The material in Chapter 14 stands alone and could be omitted or covered only partially. The material on electroweak unification and the tests of the Standard Model, presented in Chapters 15 and 16, represents one of the highlights of modern particle physics and should be considered as core. The chapter describing the Higgs mechanism is (necessarily) quite involved and it would be possible to focus solely on the properties of the Higgs boson and its discovery, rather than the detailed derivations.

Fully worked solutions to all problems are available to instructors, and these can be found at [www.cambridge.org/MPP](http://www.cambridge.org/MPP). In addition, to aid the preparation of new courses, PowerPoint slides covering most of the material in this book are available at the same location, as are all of the images in this book.

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