Chapter 2: Using and Applying Mathematics in Education

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ABSTRACT

In the mid eighties, mathematics educators propagating the teaching of mathematics by applications represented a small and unique group. Emphasizing the value of necessity of applications in mathematics education was judged as theoretical and practical action against the dominance of pure mathematics in schools raised by the new math movement. At present, in 1996, this 'movement' towards more applications has gained quite some momentum (Keitel 1993). The aim of this chapter is to establish a reasonably accurate picture of the debate and actual state of using and applying mathematics in schools.

We start a historical reflection on the ongoing deals with the arguments for applications in mathematics education. Different aspects of using and applying mathematics are dealt with detail. The case studies show also different ways in which progress is made in implementing more applications oriented curricula. We reach a rather positive outlook in the final reflection but recognize several obstacles and problems that are in the way of this positive outlook.

1. AIM OF CHAPTER

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2. PURE VS. APPLIED MATHEMATICS; SOME HISTORICAL NOTES

2.1 Pure vs. Applied Mathematics

The discussion of the dichotomy 'Pure' vs 'Applied' mathematics is interesting and sometimes emotional and seemingly never-ending. The disagreement starts already with Euclid. Although it may look strange, even Euclid's Elements have recently been claimed to be applied mathematics. Not surprisingly, this was done by the 'Champion of Applications' Kline (Alexanderson 1985). In Kline's (1980) own words:

> The real goal was the study of nature. In so far as the study of the physical world was concerned, even the truths of geometry were highly significant. It was clear to the Greeks that geometric principles were embodied in the entire structure of the universe, of which space was the primary component. (p. 24)

The partisans of the 'pure' point of view find an advocate in the work of Proclus (410-485), who formulated the then prevailing view on the philosophy behind Euclid's Elements. Although Proclus – a Neoplatonist – recognizes the practical side of the elements, the principles are not to be found in the applications but in the 'beauty and ordering of mathematical reasoning'. This seems to be exactly the reason why mathematicians love mathematics. If Kline is the Champion of Applied Mathematics, Hardy (1967) will have a good chance of being the Champion of Pure Mathematics. His near famous quote makes clear why:

> I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly, for good or ill, the least difference to the amenity of the world... Judged by all practical standards, the value of my mathematical life is nil; and outside mathematics it is trivial anyhow.
> (p. 150)

Society tolerates mathematics because of this practical importance. Greco-Roman technology certainly required more mathematics than did that of Babylon and Egypt previously, but it was still a poorly applied mathematics and this hampered the development of pure mathematics.

Mathematics in the Roman time was almost identical to astrology; it continued to play a modest role in society until the 16th century. According to Freudenthal (1973), this growth was prepared by the fact that between 1200 and 1500 more things were invented than ever before in human history – a chain of very fundamental inventions in which the art of printing is the last link.

The same attitude of mind which concocts inventions, plays tricks on nature, and searches for the secrets in numbers and figures caused this sudden growth. Mathematics was used more and more.

A large number of textbooks with often a practical touch were written. It was around 1713-1715 that the trend towards useful mathematics was concretized in new 'Elements': Christian Wolf's Elementa Matheseos Universae.

According to Bos (1983) the main difference between this and the 'Elements' of Euclid lies in the fact that Wolf accepts very clearly applications as part of mathematics; this was the generally accepted idea about mathematics in the 17th and 18th century. But Euclid and Wolf agree on one point: primarily it is not the contents of the theorem that are important, but the method and style of reasoning. Although Bos argues that mathematics is at its 'widest' in the 17th and 18th century, the really great influence of applications on the problems of mathematics dates from the beginning of the 19th century. Freudenthal (1973) sees a key role for Fourier, Poisson and Cauchy in the shift towards applications that turned out to be enormously beneficial for the development of mathematics in the 18th century.

But, too, in this century pure mathematics was created. The vast expansion of mathematics and science made it more and more difficult to be at home in both fields. The idea that mathematics is not a body of truths about nature was another source that altered radically the mathematicians' attitude towards their own work. The idea arose that it was not necessary to undertake problems of the real world. Abstract mathematics would surely prove useful.

Abstraction, generalization, and above all, specialization are three types of activity undertaken by pure mathematicians. The divergence from 'reality', the study of mathematics for its own sake provoked much discussion almost directly from the beginning. Many physicists and mathematicians – like Lord Kelvin (1867), Klein (1895), Poincaré (1905), Courant (1924) – warned against the danger of mathematics becoming more and more isolated. As Von Neumann stated in his essay The Mathematician (1947):

> As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired from 'reality' it is beset with very grave dangers. It becomes more and more pure aestheticizing, more and more purely l'art pour l'art. (p. 183)

Although completely inspired by Von Neumann, and admitting that virtually all of mathematics is rooted in the physical world, Halmos not only sees a strong dichotomy between mathematics and applied mathematics, but also regards mathematics as 'good' and applied mathematics as 'bad' (Albers 1985). Or, more precisely:

> Mathematics is security. Certainty, Truth, Beauty, Insight, Structure, Architecture. Applied Mathematics is much too often Bad, Ugly, Badly

> Arranged, Sloppy, Untrue, Undigested, Unorganized, and Unarchitected mathematics. (p. 127)

The theory of structures, received a great impetus from the work of N. Bourbaki. This group of French mathematicians published a series of books, again with the title: Eléments de Mathématique' (1939, 1949 – 58).

This structuralist approach, based on set theory, was of great influence after the Second World War. But when the influence spread to primary and secondary education, via the 'new math' and the introduction of set theory, the amount of criticism increased.

Eventually this criticism has had considerable effect. Dieudonné (1982) advocated the introduction of some structuralist aspects into secondary education at the Royaumont Conference in 1959. Some 20 years later he seems to make a correction:

> Never has Bourbaki given any opinions about the possibilities or the desirability of implementing the concepts written in this book at a lower level, and certainly not at primary or secondary schools. People, proposing such aberrations have often, to say it politely, a poor knowledge of modern mathematics. (p. 620)

In general, one can state that in the first half of this century pure mathematics stood higher than applied mathematics. Or, as Davis and Hersh put it: 'the mental universe stands higher than the physical universe' (Dieudonné 1970). This leads to more and more pure mathematics, which in turn leads to critical remarks from people outside the mathematical scene. In their isolated ivory tower they do not seem responsible to society and avoid answering questions about the meaning of their work. As E. Bishop (1984) once said in a discussion to Kahane:

> Most mathematicians feel that mathematics has meaning, but it bores them to try to find out what it is. (p. 271)

But, in recent decades, there seems to be a change in attitude. Not only is there a trend towards unification in mathematics, but there is also the feeling that true applied mathematics may be an art as well. Or, as Hilton (1976) says:

> There are many examples of the art of applied mathematics, and we see that it is correct to speak of the science of mathematics, and of the art of applied mathematics. For, since mathematics incorporates a systematic body of knowledge and involves cumulative reasoning and understanding, it is to that extent a science. And since applied mathematics involves choices which must be made on the basis of experience, intuition, and even inspiration, it partakes of the quality of

art. Thus in mathematics there are certainly to be found both art and science, and there is science in both pure and applied mathematics, as there is art. (p. 95)

There are other factors that have contributed towards a more positive attitude towards applied mathematics. The rise of information technology has made more and more realistic applications accessible for treatment by mathematical tools, and has helped in developing more tools, which in turn opened up new applications, like cryptography.

Social and societal support for more applications has also risen because of the fact that in more disciplines than ever before where mathematics has become a useful tool – social sciences in general, antropology, archeology, and so on. It may sometimes be a bit 'badly arranged, ugly, sloppy and unorganized' (Halmos) but it has proven to be successful.

Social need and relevance and technological requirements drive the development and transmission of mathematical knowledge, thus indicating that applied mathematics is the pre-eminent for mathematical growth in society. As indicated before we consider the dichotomy 'Pure' vs 'Applied' as a false one, and much of the discussion not very fruitful.

2.2 Pure vs. Applied in Education

During the last 80 years there has continuously been discussion about the desirability of including applications in mathematics education. And certainly during recent decades there is an obvious trend in literature towards more applications, although Blum (1983) doubted whether one could speak of a similar trend in the actual classroom situation.

Pollak (1976) also notices a worldwide drive to make mathematics in the school more applied. He adds that in some cases, the applied point of view is a more accurate reflection of the country's social system. 'What we demand', wrote Chairman Mao-Tse-Tung, 'is the unity of politics and art.'

In some cases it is part of a reaction against the 'new math', as we can see in Kline's (1978) Why Johnny Can't Add. In others it is a recognition of the increased mathematization of many other fields than physics. Many countries have found that the problem of motivating students becomes easier when applications are used as one of the possible motivations.

A final argument is that more and more real problems become available for use in education.

Bell (1983) believes that progress is now possible where very little progress has been made in eighty years or more because of the powerful factors mentioned above. But his worldwide survey shows that Blum may be correct when stating that in classroom practice this progress has yet to be